Day 3A Panel (linear)

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Frontiers in Econometrics
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Based on A. Colin Cameron and Pravin K. Trivedi (2009, 2010), Microeconometrics using Stata (MUS), Stata Press. and A. Colin Cameron and Pravin K. Trivedi (2005), Microeconometrics: Methods and Applications (MMA), C.U.P.

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1. Introduction

- Panel data are repeated measures on individuals (i) over time (t).
 - ▶ Regress y_{it} on \mathbf{x}_{it} for i = 1, ..., N and t = 1, ..., T.
- Complications compared to cross-section data:
 - I. Inference: correct (inflate) standard errors.
 This is because each additional year of data is not independent of previous years.
 - ▶ 2. Modelling: richer models and estimation methods are possible with repeated measures.
 - Fixed effects and dynamic models are examples.
 - ▶ 3. Methodology: different areas of applied statistics may apply different methods to the same panel data set.



- Focus on panel methods most commonly used by microeconometricians.
- Three specializations to general panel methods:
 - ▶ 1. Short panel: data on many individual units and few time periods. Then data viewed as clustered on the individual unit. Many panel methods also apply to clustered data such as cross-section individual-level surveys clustered at the village level.
 - 2. Causation from observational data: use repeated measures to estimate key marginal effects that are causative rather than mere correlation.
 - Fixed effects: assume time-invariant individual-specific effects. IV: use data from other periods as instruments.
 - ▶ 3. Dynamic models: regressors include lagged dependent variables.

Outline

- Introduction
- ② Data example: wages
- Unear models overview
- Standard linear short panel estimators
- Further topics

2. Data Example: Wages

- PSID wage data 1976-82 on 595 individuals. Balanced.
- Source: Baltagi and Khanti-Akom (1990).
 [Corrected version of Cornwell and Rupert (1998).]
- Goal: estimate causative effect of education on wages.
- Complication: education is time-invariant in these data.
 Rules out fixed effects estimation.
 Need to use IV methods (Hausman-Taylor).

- Commands describe, summarize and tabulate confound cross-section and time series variation.
- Instead use specialized panel commands after xtset:
 - xtdescribe: extent to which panel is unbalanced
 - xtsum: separate within (over time) and between (over individuals)
 variation
 - xttab: tabulations within and between for discrete data e.g. binary
 - xttrans: transition frequencies for discrete data
 - xtline: time series plot for each individual on one chart
 - xtdata: scatterplots for within and between variation.

Reading in panel data

- Data organization may be
 - ▶ long form: each observation is an individual-time (i, t) pair
 - wide form: each observation is data on i for all time periods
 - wide form: each observation is data on t for all individuals
- xt commands require data in long form
 - use reshape long command to convert from wide to long form.
- Data here are already in long form

- * Read in data set
- . use mus08psidextract.dta, clear
 (PSID wage data 1976-82 from Baltagi and Khanti-Akom
 (1990))



PSID wage data 1976-82 from Baltagi and Khanti-Akom

Summarize data using non-panel commands

- . * Describe dataset
- . describe

Contains data from mus08psidextract.dta

obs: 4,165 vars: 15

vars: 15 16 Aug 2007 16:29 size: 283.220 (97.5% of memory free) (dta has notes)

variable name	storage type	display format	value label	variable label
exp	float	%9.0q		years of full-time work experience
wks	float	%9.0g		weeks worked
occ	float	%9.0g		occupation; occ==1 if in a blue-collar occupation
ind	float	%9.0g		industry; ind==1 if working in a manufacturing indu
south	float	%9.0g		residence; south==1 if in the South area
smsa	float	%9.0g		smsa==1 if in the Standard metropolitan statistical
ms	float	%9.0g		marital status
fem	float	%9.0g		female or male
union	float	%9.0g		if wage set be a union contract
ed	float	%9.0g		years of education
blk	float	%9.0g		black
lwage	float	%9.0g		log wage
id	float	%9.0g		.5 .5.
t	float	%9.0g		
exn2	float	%9 . Oa		

- Summary statistics combine variation over i and t.
- Summarize dataset
- summarize

Variable	Obs	Mean	Std. Dev.	Min	Max
exp wks occ ind south	4165 4165 4165 4165 4165	19.85378 46.81152 .5111645 .3954382 .2902761	10.96637 5.129098 .4999354 .4890033 .4539442	1 5 0 0	51 52 1 1
smsa ms fem union ed	4165 4165 4165 4165 4165	.6537815 .8144058 .112605 .3639856 12.84538	.475821 .3888256 .3161473 .4812023 2.787995	0 0 0 0 0 4	1 1 1 1 1
blk lwage id t exp2	4165 4165 4165 4165 4165	.0722689 6.676346 298 4 514.405	.2589637 .4615122 171.7821 2.00024 496.9962	0 4.60517 1 1	1 8.537 595 7 2601

• Since 4165 (= 7×595) observations for all variables the dataset is balanced and complete.

- Listing the first few observations is useful
 - . * Organization of data set
 - . list id t exp wks occ in 1/3, clean

	id	t	exp	wks	occ
1.	1	1	3	32	0
2.	1	2	4	43	0
3.	1	3	5	40	0

• Data are in long form, sorted by id and then by t

Summarize data using panel commands

- xtset command defines i and t.
 - Allows use of panel commands and some time series operators

- * Declare individual identifier and time identifier
- . xtset id t

panel variable: id (strongly balanced)

time variable: t, 1 to 7

delta: 1 unit

 xtdescribe command summarizes number of time periods each individual is observed.

```
* Panel description of data set
xtdescribe
      id: 1, 2, ..., 595
                                                                          595
          1, 2, ..., 7
                                                               T =
           Delta(t) = 1 unit
           Span(t) = 7 periods
           (id*t uniquely identifies each observation)
Distribution of T_i:
                       min
                                 5%
                                        25%
                                                  50%
                                                             75%
                                                                     95%
                                                                             max
     Frea.
            Percent
                                Pattern
                       Cum.
                               1111111
      595
             100.00
                     100.00
```

XXXXXXX

 Data are balanced with every individual i having 7 time periods of data

100.00

595

- xtsum command splits overall variation into
 - **between variation:** variation in $\bar{x}_i = T_i^{-1} \sum_i x_{it}$ across individuals
 - within variation: variation in x_{it} around \bar{x}_i
 - . * Panel summary statistics: within and between variation
 - . xtsum lwage exp ed t

Variabl	e	Mean	Std. Dev.	Min	Max	Observa	ations
lwage	overall between within	6.676346	.4615122 .3942387 .2404023	4.60517 5.3364 4.781808	8.537 7.813596 8.621092	N = n = T =	4165 595 7
exp	overall between within	19.85378	10.96637 10.79018 2.00024	1 4 16.85378	51 48 22.85378	N = n = T =	4165 595 7
ed	overall between within	12.84538	2.787995 2.790006 0	4 4 12.84538	17 17 12.84538	N = n = T =	4165 595 7
t	overall between within	4	2.00024 0 2.00024	1 4 1	7 4 7	N = n = T =	4165 595 7

 For time-invariant variable ed the within variation is zero. For individual-invariant variable t the between variation is zero. For lwage the within variation < between variation.

- xttab command provides more detail for discrete-valued variable.
 - . * Panel tabulation for a variable
 - . xttab south

	Ove	erall	Bet	ween	Within
south	Freq.	Percent	Freq.	Percent	Percent
0	2956 1209	70.97 29.03	428 182	71.93 30.59	98.66 94.90
Total	4165	100.00	610 (n = 595)	102.52	97.54

• 29.03% on average were in the south.

30.59% were ever in the south.

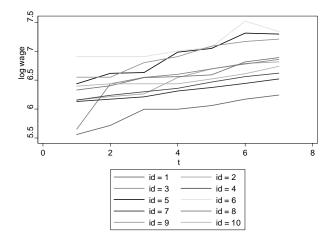
94.9% of those ever in the south were always in the south.

- xttab provides transition probabilities for discrete-valued variable.
 - . * Transition probabilities for a variable
 - . xttrans south, freq

residence; south==1 if in the South area	residence; if in the S 0		Total
0	2,527	8	2,535
	99.68	0.32	100.00
1	8	1,027	1,035
	0.77	99.23	100.00
Total	2,535	1,035	3,570
	71.01	28.99	100.00

• For the 28.99% of the sample ever in the south, 99.23% remained in the south the next period.

- 2. Data Exampl
- . * Time series plots of log wage for first 10 individuals
- . xtline lwage if id $\!<$ =10, overlay



• Much autocorrelation in each person's wage.

- Can compute autocorrelations for a variable.
 - . * First-order autocorrelation in a variable
 - . sort id t
 - . correlate lwage L.lwage L2.lwage L3.lwage L4.lwage L5.lwage L6.lwage (obs=595)

	lwage	L. Iwage	L2. Iwage	L3. Iwage	L4. Iwage	L5. lwage	L6. lwage
lwage							
-Ī.	1.0000						
L1.	0.9238	1.0000					
L2.	0.9083	0.9271	1.0000				
L3.	0.8753	0.8843	0.9067	1.0000			
L4.	0.8471	0.8551	0.8833	0.8990	1.0000		
L5.	0.8261	0.8347	0.8721	0.8641	0.8667	1.0000	
L6.	0.8033	0.8163	0.8518	0.8465	0.8594	0.9418	1.0000

- High serial correlation: $Cor[y_t, y_{t-6}] = 0.80$.
- Note that estimated autocorrelations without imposing stationarity.

Pooled OLS

- Pooled OLS is regular OLS of y_{it} on \mathbf{x}_{it} .
 - . * Pooled OLS with incorrect default standard errors
 - . regress lwage exp exp2 wks ed, noheader

exp2 0007156 .0000528 -13.56 0.000 0008191 0006121 wks .005827 .0011827 4.93 0.000 .0035084 .0081456 ed .0760407 .0022266 34.15 0.000 .0716754 .080406	lwag	e	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
	exp wk e	2 s d	0007156 .005827 .0760407	.0000528 .0011827 .0022266	-13.56 4.93 34.15	0.000 0.000 0.000	0008191 .0035084 .0716754	.0493663 0006121 .0081456 .080406 5.039963

- The default standard errors erroneously assume errors are independent over *i* for given *t*.
 - ▶ Reason: Assumes more information content from data then is the case.

Should instead use cluster-robust standard errors

- * Pooled OLS with cluster-robust standard errors
- . regress lwage exp exp2 wks ed, noheader vce(cluster id) (Std. Err. adjusted for 595 clusters in id)

lwage	Coef.	Robust Std. Err.	t	P> t	[95% Conf.	Interval]
exp	.044675	.0054385	8.21	0.000	.0339941	.055356
exp2	0007156	.0001285	-5.57	0.000	0009679	0004633
wks	.005827	.0019284	3.02	0.003	.0020396	.0096144
ed	.0760407	.0052122	14.59	0.000	.0658042	.0862772
_cons	4.907961	.1399887	35.06	0.000	4.633028	5.182894

- Cluster-robust standard errors are twice as large as default! Cluster-robust t-statistics are half as large as default!
- Typical result. Need to use cluster-robust se's if use pooled OLS.

3. Linear panel models: Basic considerations

- Regular time intervals assumed.
- Unbalanced panel okay (xt commands handle unbalanced data). [Should then rule out selection/attrition bias].
- ③ Short panel assumed, with T small and $N \to \infty$. [Versus long panels, with $T \to \infty$ and N small or $N \to \infty$.]
- Errors are correlated.[For short panel: correlated over t for given i, but not over i.]
- Parameters may vary over individuals or time. Intercept: Individual-specific effects model (fixed or random effects). Slopes: Pooling and random coefficients models.
- Regressors: time-invariant, individual-invariant, or vary over both.
- Prediction: ignored.
 [Not always possible even if marginal effects computed.]
- Dynamic models: possible.[Usually static models are estimated.]



Basic linear panel models

Pooled model (or population-averaged)

$$y_{it} = \alpha + \mathbf{x}'_{it}\boldsymbol{\beta} + u_{it}. \tag{1}$$

ullet Two-way effects model allows intercept to vary over i and t

$$y_{it} = \alpha_i + \gamma_t + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}. \tag{2}$$

Individual-specific effects model

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}, \tag{3}$$

where α_i may be fixed effect or random effect.

Mixed model or random coefficients model allows slopes to vary over i

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta}_i + \varepsilon_{it}. \tag{4}$$

Fixed effects versus random effects model

Individual-specific effects model:

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + (\alpha_i + \varepsilon_{it}).$$

- Fixed effects (FE):
 - \triangleright α_i is a random variable possibly correlated with \mathbf{x}_{it}
 - so regressor \mathbf{x}_{it} may be endogenous (wrt to α_i but not ε_{it}) e.g. education is correlated with time-invariant ability
 - \triangleright pooled OLS, pooled GLS, RE are inconsistent for β
 - within (FE) and first difference estimators are consistent.
- Random effects (RE) or population-averaged (PA):
 - α_i is purely random (usually iid $(0, \sigma_{\alpha}^2)$) unrelated to \mathbf{x}_{it}
 - ightharpoonup so regressor $m f x_{it}$ is exogenous
 - \blacktriangleright all estimators are consistent for $\pmb{\beta}$
- Fundamental divide: microeconometricians FE versus others RE.

4. Linear panel estimators: cluster-robust inference

- There are many different panel estimators detailed below.
- Many methods assume ε_{it} and α_i (if present) are iid.
 - ▶ This yields wrong standard errors if heteroskedastic or if errors are not equicorrelated over time for a given individual.
- Instead, for a short panel can relax assumptions and use cluster-robust inference.
 - This allows heteroskedasticity and general correlation over time for given i.
 - ▶ Independence over *i* is still assumed.
- In Stata
 - For xtreg use option vce(robust) does cluster-robust
 - For some other xt commands use option vce(cluster)
 - And for some other xt commands there is no option, but may be able to do a cluster bootstrap.

Fixed effects estimator

• Mean-differencing eliminates α_i

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}$$

$$\Rightarrow \quad \bar{y}_i = \alpha_i + \bar{\mathbf{x}}'_i\boldsymbol{\beta} + \bar{\varepsilon}_i$$

$$\Rightarrow \quad (y_{it} - \bar{y}_i) = (\mathbf{x}_{it} - \bar{\mathbf{x}}_i)'\boldsymbol{\beta} + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

- The within or fixed effects estimator is OLS of $(y_{it} \bar{y}_i)$ on $(\mathbf{x}_{it} \overline{\mathbf{x}}_i)$
 - Efficiency loss as use only within variation
 - ▶ Coefficient of any time-invariant regressor is not identified $(x_{it} = \overline{x}_i)$
 - Use cluster-robust standard errors
 - Stata command xtreg, fe
- This can be shown to be same as OLS of y_{it} on N individual dummies and \mathbf{x}_{it}
 - least squares dummy variable (LSDV) estimator.

Within or FE estimates:

```
. * Within or FE estimator with cluster-robust standard errors
```

. xtreg lwage exp exp2 wks ed, fe vce(robust)

Fixed-effects (within) regression	Number of obs	=	4165
Group variable: id	Number of groups		595
R-sq: within = 0.6566	Obs per group: min	=	7
between = 0.0276	avg		7.0
overall = 0.0476	max		7
corr(u_i, xb) = -0.9107	F(3,594) Prob > F	=	1059.72

(Std. Err. adjusted for 595 clusters in id)

lwage	Coef.	Robust Std. Err.	t	P> t	[95% Conf.	. Interval]
exp exp2 wks ed _cons	.1137879 0004244 .0008359 (dropped) 4.596396	.0040289 .0000822 .0008697	28.24 -5.16 0.96 76.49	0.000 0.000 0.337	.1058753 0005858 0008721 4.478384	.1217004 0002629 .0025439 4.714408
sigma_u sigma_e rho	1.0362039 .15220316 .97888036	(fraction	of varia	nce due t	o u_i)	

• Variable ed is not identified as time-invariant regressor in this dataset.

4□ → 4周 → 4 = → 4 = → 9 Q P

Random effects estimator

Random effects estimator is FGLS estimator for the RE model

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}$$

 $\alpha_i \sim \text{i.i.d.}[0, \sigma^2_{\alpha}]$
 $\varepsilon_{it} \sim \text{i.i.d.}[0, \sigma^2_{\varepsilon}]$

This can be shown to equal OLS in the transformed model

$$(y_{it} - \widehat{ heta}_i ar{y}_i) = (\mathbf{x}_{it} - \widehat{ heta}_i ar{\mathbf{x}}_i)' oldsymbol{eta} + ext{error},$$

where $\hat{\theta}_i$ is a consistent estimate of $\theta_i = 1 - \sqrt{\sigma_{\varepsilon}^2/(T_i\sigma_{\alpha}^2 + \sigma_{\varepsilon}^2)}$.

Random effects estimates:

- . * Random effects estimator with cluster-robust standard errors
- . xtreg lwage exp exp2 wks ed, re vce(robust) theta

```
Random-effects GLS regression
                                               Number of obs
                                                                        4165
Group variable: id
                                               Number of groups =
                                                                         595
R-sq: within = 0.6340
                                               Obs per group: min =
      hetween = 0.1716
                                                              ava =
                                                                          7.0
      overall = 0.1830
                                                              max =
Random effects u i ~ Gaussian
                                               Wald chi2(5)
                                                                 = 175914.07
corr(u_i, X) = 0  (assumed)
                                               Prob > chi2
                                                                       0.0000
theta
                 = .82280511
```

(Std. Err. adjusted for clustering on id)

lwage	Coef.	Robust Std. Err.	z	P> z	[95% Conf.	Interval]
exp exp2 wks ed _cons	.0888609 0007726 .0009658 .1117099 3.829366	.0028515 .0000623 .0009286 .0063256 .1039432	31.16 -12.40 1.04 17.66 36.84	0.000 0.000 0.298 0.000 0.000	.0832721 0008946 0008542 .099312 3.625641	.0944497 0006505 .0027857 .1241079 4.033091
sigma_u sigma_e rho	.31951859 .15220316 .81505521	(fraction	of varia	nce due t	co u_i)	

ullet Option theta gives $\widehat{ heta} = 0.82 = 1 - \sqrt{0.152^2/(7 imes 0.319^2 + 0.152^2)}$.

Fixed effects versus random effects estimators

- RE has advantages that can estimate all parameters and may be more efficient.
 - But RE is inconsistent if fixed effects present.
- Use Hausman test to discriminate between FE and RE.
 - ► This tests difference between FE and RE estimates is statistically significantly different from zero.
- Do not use hausman command as it requires RE estimator fully efficient.
- Do use a panel bootstrap of the Hausman test.
- Or use the Wooldridge (2002) robust version of Hausman test.
 - ▶ Test $H_0: \gamma = \mathbf{0}$ in the auxiliary OLS regression

$$(y_{it} - \widehat{\theta}\bar{y}_i) = (1 - \widehat{\theta})\alpha + (\mathbf{x}_{it} - \widehat{\theta}\mathbf{x}_i)'\beta + (\mathbf{x}_{1it} - \mathbf{x}_{1i})'\gamma + v_{it},$$

where \mathbf{x}_1 denotes time-varying regressors only.

Use cluster-robust standard errors for this test.

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Other panel estimators

- Population-averaged or pooled GLS estimator
 - ▶ Regress y_{it} on \mathbf{x}_{it} using feasible GLS as error u_{it} is not iid.
 - **Example** is to assume that u_{it} is an AR(2) error.
 - * xtgee lwage exp exp2 wks ed, corr(ar 2) vce(robust)
- Between estimator
 - ▶ OLS regression of \bar{y}_i on $\bar{\mathbf{x}}_i$, i.e. use each individual's averages.
 - Uses only between variation in the data.
 - * xtreg lwage exp exp2 wks ed, be
- First differences estimator
 - ▶ OLS regression of $(y_{it} y_{i,t-1})$ on $(\mathbf{x}_{it} \mathbf{x}_{i,t-1})$, i.e. use first differences.
 - First-differencing eliminates α_i in $y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}$
 - So like within it is consistent in FE model
 - ★ regress D.(lwage \$xlist), vce(cluster id)



Summary of standard linear panel estimators

- Pooled OLS (command regress)
- Pooled GLS (command xtgee)
- Between estimator (command xtreg, be)
- Within or FE estimator (command xtreg, fe)
- Random effects estimator (command xtreg, re)
- First differences estimator (command regress with d.y and d.x)

Estimator comparison

- . * Compare various estimators (with cluster-robust se's)
- . global xlist exp exp2 wks ed
- . quietly regress lwage \$xlist, vce(cluster id)
- estimates store OLS
- . quietly xtgee lwage exp exp2 wks ed, corr(ar 2)
 vce(robust)
- . estimates store PFGLS
- . quietly xtreg lwage \$xlist, be
- . estimates store BE
- . quietly xtreg lwage \$xlist, re vce(robust)
- . estimates store RE
- . quietly xtreg lwage \$xlist, fe vce(robust)
- . estimates store FE
- . estimates table OLS PFGLS BE RE FE, b(%9.4f) se stats(N)

Variable	OLS	PFGLS	BE	RE	FE
exp	0.0447	0.0719	0.0382	0.0889	0.1138
	0.0054	0.0040	0.0057	0.0029	0.0040
exp2	-0.0007	-0.0009	-0.0006	-0.0008	-0.0004
	0.0001	0.0001	0.0001	0.0001	0.0001
wks	0.0058	0.0003	0.0131	0.0010	0.0008
	0.0019	0.0011	0.0041	0.0009	0.0009
ed	0.0760	0.0905	0.0738	0.1117	0.0000
	0.0052	0.0060	0.0049	0.0063	0.0000
_cons	4.9080	4.5264	4.6830	3.8294	4.5964
	0.1400	0.1057	0.2101	0.1039	0.0601
N	4165.0000	4165.0000	4165.0000	4165.0000	4165.0000

legend: b/se

- Coefficients vary considerably across OLS, RE, FE and RE estimators.
 - ▶ FE and RE similar as $\widehat{\theta} = 0.82 \simeq 1$.
- Not shown is that even for FE and RE cluster-robust changes se's.
- Coefficient of ed not identified for FE as time-invariant regressor!

5. Further Topics

- Long panels not covered here
 - ▶ Asymptotics are in $T \to \infty$ with N fixed or $N \to \infty$
- Instrumental variables
- Estimators include panel IV and Hausman-Taylor
- Dynamic short panels $y_{it} = \alpha_i + \rho y_{i,t-1} + \mathbf{x}'_{it} \boldsymbol{\beta} + u_{it}$
 - Usual FE and FD estimates are inconsistent
 - ▶ Instead use Arellano-Bond (panel IV in FD model with lagged $y_{i,t-k}$ as instruments)
- Random coefficients
 - mixed linear models
- Nonlinear panel models
 - rightharpoonup cannot always eliminate fixed effects α_i (only in Poisson, logit, negative binomial)
 - leads to alternatives such as correlated random effects.



6. Stata Commands

Linear panel estimators

```
Panel summary
                xtset; xtdescribe; xtsum; xtdata;
                xtline; xttab; xttran
Pooled OLS
                regress
Feasible GLS
                xtgee, family(gaussian)
                xtgls; xtpcse
Random effects
                xtreg, re; xtregar, re
Fixed effects
                xtreg, fe; xtregar, fe
Random slopes
                xtmixed; quadchk; xtrc
First differences
                regress (with differenced data)
Static IV
                xtivreg; xthtaylor
Dynamic IV
                xtabond; xtdpdsys; xtdpd
```

Nonlinear panel estimators

Estimator	Count data	Binary data
Pooled	poisson	logit
	nbreg	probit
GEE (PA)	xtgee,family(poisson)	xtgee,family(binomial) lin
	xtgee,family(nbinomial)	xtgee,family(poisson) link
RE	xtpoisson, re	xtlogit, re
	xtnbreg, fe	xtprobit, re
Random slopes	xtmepoisson	xtmelogit
FE	xtpoisson, fe	xtlogit, fe
	xtnbreg, fe	

- Also tobit and xttobit for Tobit models
 - xttobit is for RE only (same for xtprobit).