BGPE Frontiers in Econometrics 2009
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Answers to Final Exam

1.(a) We have $\widehat{\boldsymbol{\beta}} = (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'(\mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\gamma} + \mathbf{v}) = \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Z}\boldsymbol{\gamma} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{v}$.
$\mathrm{E}[\widehat{\boldsymbol{\beta}}] = \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Z}\boldsymbol{\gamma} +$ $\mathrm{E}[(\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{v}] = \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Z}\boldsymbol{\gamma}$
using $\mathbf{Z}$, $\mathbf{A}$ and nonstochastic and $\mathrm{E}[\mathbf{v}] = \mathbf{0}$.

(b) $\mathrm{V}[\widehat{\boldsymbol{\beta}}] = \mathrm{E}[(\widehat{\boldsymbol{\beta}} - \mathrm{E}[\widehat{\boldsymbol{\beta}}])(\widehat{\boldsymbol{\beta}} - \mathrm{E}[\widehat{\boldsymbol{\beta}}])'] = \mathrm{E}[((\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{v})((\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{v})'] = (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\Omega\mathbf{Z}(\mathbf{X}'\mathbf{Z})^{-1}$
using $\mathbf{Z}$, $\mathbf{A}$ and nonstochastic and $\mathrm{V}[\mathbf{v}] = \Omega$.

(c) We have $\widehat{\boldsymbol{\beta}} = \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Z}\boldsymbol{\gamma} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{v}$. So

$$\begin{aligned}
\widehat{\boldsymbol{\beta}} &= \boldsymbol{\beta} + \left(N^{-1}\mathbf{Z}'\mathbf{X}\right)^{-1} N^{-1}\mathbf{Z}'\mathbf{Z}\boldsymbol{\gamma} + \left(N^{-1}\mathbf{Z}'\mathbf{X}\right)^{-1} N^{-1}\mathbf{Z}'\widehat{\boldsymbol{\beta}} \\
&\xrightarrow{p} \boldsymbol{\beta} + \left(\operatorname{plim} N^{-1}\mathbf{Z}'\mathbf{X}\right)^{-1} \operatorname{plim} N^{-1}\mathbf{Z}'\mathbf{Z}\boldsymbol{\gamma} + \left(\operatorname{plim} N^{-1}\mathbf{Z}'\mathbf{X}\right)^{-1} \operatorname{plim} N^{-1}\mathbf{Z}'\mathbf{v} \\
&\xrightarrow{p} \boldsymbol{\beta} \ + \left(\operatorname{plim} N^{-1}\mathbf{Z}'\mathbf{X}\right)^{-1} \operatorname{plim} N^{-1}\mathbf{Z}'\mathbf{Z}\boldsymbol{\gamma}
\end{aligned}$$

given plims exist and are finite nonzero and $\operatorname{plim} N^{-1}\mathbf{Z}'\mathbf{v} = \mathbf{0}$.

(d) Use White type estimate

$$\begin{aligned}
\widehat{\mathrm{V}}[\widehat{\boldsymbol{\beta}}] &= (\mathbf{Z}'\mathbf{X})^{-1}(\mathbf{Z}'\mathrm{Diag}[\widehat{v}_i^2]\mathbf{Z})(\mathbf{X}'\mathbf{Z})^{-1} \\
&= \left(\textstyle\sum_i \mathbf{z}_i\mathbf{x}_i'\right)^{-1}\left(\textstyle\sum_i \widehat{v}_i^2\mathbf{z}_i\mathbf{z}_i'\right)^{-1}\left(\textstyle\sum_i \mathbf{x}_i\mathbf{z}_i'\right)^{-1}
\end{aligned}$$

where $\widehat{v}_i = y_i - \mathbf{x}_i'\widehat{\boldsymbol{\beta}}$.

(e) A central limit theorem requires stronger assumptions.

2. For parts (a)-(d) consider the following model.
Consider the panel data model

$$y_{it} = \alpha_i + \mathbf{w}_i'\boldsymbol{\gamma} + \mathbf{x}_{it}'\boldsymbol{\beta} + \varepsilon_{it},$$

where $\alpha_i \sim$ i.i.d. $[0, \sigma_\alpha^2]$ and $\varepsilon_{it} \sim$ i.i.d. $[0, \sigma_\alpha^2]$.

(a) OLS is consistent. But default standard errors are incorrect as they ignore correlation of the error over t for given i (since $\mathrm{Cov}[\alpha_i + \varepsilon_{it},\, \alpha_i + \varepsilon_{is}] = \mathrm{Cov}[\alpha_i, \alpha_i] = \sigma_\alpha^2 \neq 0$.

(b) FE estimator is consistent. And default FE standard errors are correct.
Only problems are efficiency loss (compared to RE) and $\boldsymbol{\gamma}$ not identified as $\mathbf{w}_i$ is time invariant.

(c) FE estimator is consistent. And default RE standard errors are correct. And RE is fully efficient (FGLS) given the above assumptions. No problems.

(d) Given the assumptions made, both RE and FE are valid estimators and we do not expect to reject given the Hausman test. So somethng is wrong with our assumptions. One possibility is that α_i is corelated with $\mathbf{w}_i$, so we should be usinghte FE modle not the RE model.

(e) GMM estimator minimizes $(\mathbf{Z}'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}))'(\mathbf{Z}'\mathbf{Z})^{-1}(\mathbf{Z}'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}))$.

(f) F.O.C. are $2\mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}(\mathbf{Z}'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) = \mathbf{0}$.

Solving $\mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{y} = \mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}(\mathbf{Z}'\mathbf{X}\boldsymbol{\beta})$
So $\widehat{\boldsymbol{\beta}} = [\mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}(\mathbf{Z}'\mathbf{X})]^{-1}\mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{y}$.

(g) No. Most efficient estimator uses weighting matrix $(\mathrm{V}[\mathbf{Z}'(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})])^{-1}$.
For heteroskedastic errors $\mathrm{V}[\mathbf{Z}'(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})] = \mathbf{Z}'\mathrm{Diag}[\sigma_i^2]\mathbf{Z}$.
So use $(\mathbf{Z}'\mathrm{Diag}[\widehat{u}_i^2]\mathbf{Z}')^{-1} = \left(\sum_i \widehat{u}_i^2\mathbf{z}_i\mathbf{z}_i'\right)^{-1}$ where $\widehat{u}_i = y_i - \mathbf{x}_i'\widehat{\boldsymbol{\beta}}$.

3.(a) Log-likelihood is

$$\begin{aligned} L_N(\boldsymbol{\beta}) &= \sum\nolimits_i y_i \ln\exp(\mathbf{x}_i'\boldsymbol{\beta}) + (1-y_i)\ln[1-\exp(\mathbf{x}_i'\boldsymbol{\beta})] \\ &= \sum\nolimits_i y_i\mathbf{x}_i'\boldsymbol{\beta} + (1-y_i)\ln[1-\exp(\mathbf{x}_i'\boldsymbol{\beta})]. \end{aligned}$$

(b) Problem that $\Pr[y_i=1] = \exp(\mathbf{x}_i'\boldsymbol{\beta}) > 1$ (and $\Pr[y_i=0] < 0$).

(c) Multinomial logit.

$$\Pr[y_i = k] = \frac{\exp(\mathbf{x}_i'\boldsymbol{\beta}_k)}{\sum_{l=1}^{3}\exp(\mathbf{x}_i'\boldsymbol{\beta}_l)}, \qquad k = 1,2,3.$$

(d) ARUM

$$U_{ki} = \mathbf{x}_i'\boldsymbol{\beta}_k + \varepsilon_{ik}$$

where ε_{ik} are i.i.d. Type I extreme value.

(e) We have

$$\begin{aligned} \mathsf{E}[y_2|y_1^* > 0] &= \mathbf{x}_2'\boldsymbol{\beta}_2 + \mathsf{E}[u_2|\mathbf{x}_1'\boldsymbol{\beta}_1 + u_1 > 0] \\ &= \mathbf{x}_2'\boldsymbol{\beta}_2 + \mathsf{E}\left[(\delta\times u_1 + v)|u_1 > -\mathbf{x}_1'\boldsymbol{\beta}_1\right] \\ &= \mathbf{x}_2'\boldsymbol{\beta}_2 + \delta\times\mathsf{E}[u_1|u_1 > -\mathbf{x}_1'\boldsymbol{\beta}_1] \\ &= \mathbf{x}_2'\boldsymbol{\beta}_2 + \delta\times\lambda(\mathbf{x}_1'\boldsymbol{\beta}_1) \end{aligned}$$

(f) Estimator solves the sample moment conditions (f.o.c)

$$4\sum\nolimits_i (y_i - \mathbf{x}_i'\boldsymbol{\beta})^3\mathbf{x}_i = \mathbf{0}.$$

For consistency, we require these to hold in the population, or

$$\mathrm{E}[(y_i - \mathbf{x}_i'\boldsymbol{\beta})^3|\mathbf{x}_i] = 0 \text{ or } \mathrm{E}[u_i^3|\mathbf{x}_i] = 0.$$

Error is symmetric conditiinal on the regressors.

4.(a) $\partial\Pr[y_i = 1|\mathbf{x}_i]/\partial\mathbf{x}_{ik} = \phi(\mathbf{x}_i'\boldsymbol{\beta})\times\beta_k$, where $\phi(\cdot)$ is the standard normal density.
Default for `mfx` is to compute the MEM: $\phi(\overline{\mathbf{x}}_i'\widehat{\boldsymbol{\beta}})\times\widehat{\beta}_k$.

(b) `margeff` computes the AME: $\frac{1}{N}\sum_i\phi(\mathbf{x}_i'\widehat{\boldsymbol{\beta}})\times\widehat{\beta}_k$.

(c) No. The coefficient are scaled quite differently.
Instead expect approximately $0.4\times\widehat{\boldsymbol{\beta}}_{\text{PROBIT}} = \widehat{\boldsymbol{\beta}}_{\text{OLS}}$.

(d) Yes. Should be after first command `probit`, can you say anything about the relative impacts of variables `private` and `chronic`?

(e) An ordered logit or probit as categorical variable with ordered outcomes.

(f) Biased estimator as mean of `b2f` has 95% confidence interval (.889, .914) that clearly does not include 1. And wrong se's as mean of `se2f` (.141) does not equal standard deviation of `b2f` (.199).

(g) The error in the `ystar` equation was $N[0,|x|^2]$ and not $N[0,1]$. Heteroskedastic.