

**Econ 102 (Analysis of Economic Data): Cameron Spring 2016**  
**Solutions to Second Midterm Exam**

**Version A**

- 1.(a)** 1. Model  $y_i = \beta_1 + \beta_2 x_i + u_i$   
2. Zero conditional mean error.  $E[u_i|x_i] = 0$  for all  $i$ .  
3. Constant conditional variance error.  $\text{Var}[u_i|x_i] = \sigma_u^2$  for all  $i$ .  
4. Independent errors.  $u_i$  independent of  $u_j$  for all  $i \neq j$ .  
**(b)** Assumption 3.  
**(c)** “Best” means smallest variance.  
**(d)** The correlation between  $x$  and  $y$  is 0.6, or  $R^2 = 0.6^2 = 0.36$  or a one standard deviation increase in  $x$  is associated with a 0.6 standard deviation change in  $y$ .  
**(e)** Type 1 error is rejecting  $H_0$  when  $H_0$  is true.

**2.(a)**  $b_2 = 54.69$  so a \$1,000 increase in newspaper advertising is associated with 54.69 more units sold.

**(b)** A 95% confidence interval for  $\beta_2$  is (22.005, 87.381) from the regression output.

**(c)** A 90% confidence interval for  $\beta_2$  is  $b_2 \pm t_{.05,198} \times s_{b_2}$   
 $= 54.6931 \pm \text{invttail}(198, .05) \times 16.575 = 54.6931 \pm 1.6526 \times 16.575 = 54.6931 \pm 27.3918 =$   
(27.301, 82.085).

**(d)**  $H_0 : \beta_2 = 0$  against  $H_a : \beta_2 \neq 0$ .

Simplest is using p-value approach:  $p = 0.001 < 0.05$ . So reject  $H_0$  at level 0.05.

Alternatively and equivalently  $|t| = 3.30 > t_{.025,198} = \text{invttail}(198, .025) = 1.972$ . So reject  $H_0$ .

Conclude there is a statistically significant relationship at level 0.05.

**(e)**  $H_0 : \beta_2 \leq 30$  against  $H_a : \beta_2 > 30$ . (Make the claim the alternative)

$t = (b_2 - 30)/s_{b_2} = (54.6931 - 30)/16.575 = 24.6931/16.575 = 1.490$ .

Since  $t = 1.490 < t_{.05,198} = \text{invttail}(198, .05) = 1.652$  we do not reject  $H_0$ .

Conclude that sales do not rise by more than 30 units for each \$1,000 increase in newspaper advertising.

**(f)**  $\hat{y} = b_1 + b_2 x = 12351.41 + 54.6931 \times 100 = \$17,821$ .

**3.(a)**  $\text{Var}[\bar{y}] = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2 = \frac{1}{9} \times 90 = 10$ .

**(b)**  $b_2 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = 20/10 = 2$ .

$b_1 = \bar{y} - b_2 \bar{x} = 20 - 2 \times 1 = 18$ .

**(c)**  $s_e^2 = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{(n-2)} = 50/8 = 6.25$ .  $s_e = \sqrt{6.25} = 2.5$ .

**(d)**  $R^2 = 1 - \text{ResidSS}/\text{TotalSS} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} = 1 - 50/90 = 4/9 = 0.4444$ .

**(e)**  $r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} = 20/\sqrt{10 \times 90} = 20/30 = 2/3$ .

(Or use  $r_{xy}^2 = R^2 = 4/9$  so  $r_{xy} = \sqrt{4/9} = 2/3$ ).

**(f)** We expect  $b_2$  on average equals  $\beta_2$  which from the code is  $\beta_2 = 3$ .

**Multiple Choice for Versions A and B**

Question	1.	2.	3.	4.	5.
Answer Version A	a	d	b	b	a
Answer Version B	b	a	b	c	b

## Version B

1.(a) 1. Model  $y_i = \beta_1 + \beta_2 x_i + u_i$

2. Zero conditional mean error.  $E[u_i|x_i] = 0$  for all  $i$ .

3. Constant conditional variance error.  $\text{Var}[u_i|x_i] = \sigma_u^2$  for all  $i$ .

4. Independent errors.  $u_i$  independent of  $u_j$  for all  $i \neq j$ .

(b) Assumption 3.

(c) "Best" means smallest variance.

(d) The correlation between  $x$  and  $y$  is 0.4, or  $R^2 = 0.4^2 = 0.16$  or a one standard deviation increase in  $x$  is associated with a 0.4 standard deviation change in  $y$ .

(e) Type 2 error is failing to reject  $H_0$  when  $H_0$  is false.

2.(a)  $b_2 = 48.01$  so a \$1,000 increase in newspaper advertising is associated with 48.01 more units sold.

(b) A 95% confidence interval for  $\beta_2$  is (13.908, 82.114) from the regression output.

(c) A 99% confidence interval for  $\beta_2$  is  $b_2 \pm t_{.005,198} \times s_{b_2}$

$= 48.0108 \pm \text{invttail}(178, .005) \times 17.281 = 48.0108 \pm 2.6037 \times 17.281 = 48.0108 \pm 44.9945 = (3.016, 93.005)$ .

(d)  $H_0 : \beta_2 = 0$  against  $H_a : \beta_2 \neq 0$ .

Simplest is using p-value approach:  $p = 0.006 < 0.05$ . So reject  $H_0$  at level 0.05.

Alternatively and equivalently  $|t| = 2.78 > t_{.025,178} = \text{invttail}(178, .025) = 1.973$ . So reject  $H_0$ .

Conclude there is a statistically significant relationship at level 0.05.

(e)  $H_0 : \beta_2 \leq 10$  against  $H_a : \beta_2 > 10$ . (Make the claim the alternative)

$t = (b_2 - 10)/s_{b_2} = (48.0108 - 10)/17.281 = 38.0108/17.281 = 2.199$ .

Since  $t = 2.199 > t_{.05,178} = \text{invttail}(178, .05) = 1.653$  we reject  $H_0$ .

Conclude that sales rise by more than 10 units for each \$1,000 increase in newspaper advertising.

(f)  $\hat{y} = b_1 + b_2 x = 12502.97 + 48.0108 \times 200 = \$22,105$ .

3.(a)  $\text{Var}[\bar{y}] = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2 = \frac{1}{9} \times 360 = 40$ .

(b)  $b_2 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = 80/40 = 2$ .

$b_1 = \bar{y} - b_2 \bar{x} = 20 - 2 \times 2 = 16$ .

(c)  $s_e^2 = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{(n-2)} = 200/8 = 25$ .  $s_e = \sqrt{25} = 5$ .

(d)  $R^2 = 1 - \text{ResidSS}/\text{TotalSS} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} = 1 - 200/360 = 4/9 = 0.4444$ .

(e)  $r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} = \frac{80}{\sqrt{40 \times 360}} = \frac{80}{120} = 2/3$ .

(Or use  $r_{xy}^2 = R^2 = 4/9$  so  $r_{xy} = \sqrt{4/9} = 2/3$ ).

(f) We expect  $b_2$  on average equals  $\beta_2$  which from the code is  $\beta_2 = 1$ .

The course grade will be based on a curve from the combined scores of midterm 1 (22.5%), midterm 2 (22.5%), final (45%) and assignments (10%). **The curve for this exam is only a guide.** Suggested average GPA for this course is 2.7. Curve below has average GPA 2.72 for this exam.

Scores out of	30	A+	29.5 and above	C+	23 and above
75th percentile	26.5 (88.3%)	A	27.5 and above	C	22.5 and above
Median	24.5 (82%)	A-	26.5 and above	C-	21.5 and above
25th percentile	23 (77%)	B+	25.5 and above	D+	20.5 and above
		B	25 and above	D	20 and above
		B-	24 and above	D-	19 and above