Econ 102 (Analysis of Economic Data): Cameron Winter 2014
Solutions to Second Midterm Exam

Version A

1.(a) $\Delta \ln y = 7.5 - 3 \simeq 4.5$; $\Delta year \simeq 65$. Growth rate in levels $\simeq 4.5/65 \simeq 0.07$ or 7% per year.
(b) $72/r$ years to double. Here $72/r = 4$ so $r = 18\%$ per annum..
(c) $(y_t + y_{t-1} + y_{t-2} + y_{t-3})/4$.
(d) FRED (Federal Reserve Bank of St. Louise economic database).
(e) A line chart with both y and x plotted against year.
(f) $\bar{x} = (2+3+5+6)/4 = 4$ and $\bar{y} = (6+1+1+4)/4 = 3$.

x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$
2	6	-2	3	-6	4
3	1	-1	-2	2	1
5	1	1	-2	-2	1
6	4	2	1	2	4

$b_2 = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) / \sum_{i=1}^n (x_i - \bar{x})^2 = -4/10 = -0.4.$ $\quad b_1 = \bar{y} - b_2\bar{x} = 3 - (-.4) \times 4 = 4.6.$

2.(a) The mean charge increases by $1000 b_2 = 1000 \times 1.314908 = 1,315$ dollars.
(b) A 95% confidence interval for β_2 is (1.0562, 1.5736) from the regression output.
(c) A 99% confidence interval for β_2 is $b_2 \pm t_{.005,.167} \times s_{b_2}$
$= 1.3149 \pm invttail(167, .005) \times .13105 = 1.3149 \pm 2.6056 \times .13105 = 1.3149 \pm .3415 = (.9734, 1.6564)$.
(d) $H_0 : \beta_2 = 0$ against $H_a : \beta_2 \neq 0$.
Using p-value approach: $p = 0.000 < 0.05$. So reject H_0.
Alternatively and equivalently $|t| = 10.03 > t_{.025,167} = invttail(167, .025) = 1.974$. So reject H_0.
Conclude there is a statistically significant relationship at level 0.05.
(e) $H_0 : \beta_2 \leq 0$ against $H_a : \beta_2 > 0$. Important: Claim is the alternative hypothesis.
First $b_2 > 0$ so on the right side for an upper-tail alternative.
Using p-value approach: $p = 0.000/2 = 0.000 < 0.05$. So reject H_0.
Alternatively and equivalently $t = 10.03 > t_{.05,167} = invttail(167, .05) = 1.654$. So reject H_0.
Conclude that agree with claim that mean charge rises with mean cost at level 0.05.
(f) $H_0 : \beta_2 = 1$ against $H_a : \beta_2 \neq 1$.
$t = (b_2 - 1)/s_{b2} = (1.3149 - 1)/.13105 = .3149/.13105 = 2.403$.
Since $|t| = 2.403 < t_{.005,167} = invttail(167, .005) = 2.606$ we do not reject H_0.
Conclude that mean charge rises by \$1 for each \$1 increase in mean cost at level 0.01.

3.(a) $\widehat{y} = b_1 + b_2 x = 20334.33 + 1.314908 \times 20000 = \$46.632$.
(b) A wrong answer (no credit) is $1/1.314908 = .7605$.
A full credit answer is that there is not enough information.
(A better answer, this is tricky, is that the slope coefficient from regress y on x is $r_{xy} \times s_y/s_x = \sqrt{.3761} \times 10376/22249 = .2860$.)
(c) This is just the sample mean of `meancharge`: 47957.27.

4.(a) R^2 = ExplainedSS / TotalSS = 40/160 = 0.25.
(b) Correlation coefficient $r_{xy} = \sqrt{R^2} = \sqrt{0.25} = 0.5$.
(c) Standard error of the residual = $\sqrt{\text{ResidualSS / (n-2)}} = \sqrt{(160-40)/8} = \sqrt{15} = 3.87$.
(d) This produces the residuals $y_i - \widehat{y}_i$.

Multiple Choice for Versions A and B

Question	**1.**	**2.**	**3.**	**4.**	**5.**
Answer Version A	*d*	*c*	*a*	*b*	*a*
Answer Version B	*a*	*c*	*a*	*d*	*b*

For 3. the key assumptions are $y = \beta_1 + \beta_2 x + u$ and $\text{E}[u|x] = 0$.

Version B

1.(a) $\Delta \ln y = 9 - 3 \simeq 6$; $\Delta year \simeq 65$. Growth rate in levels $\simeq 6/65 \simeq 0.09$ or 9% per year.
(b) $72/r$ years to double. Here $72/r = 6$ so $r = 12\%$ per annum.
(c) $(y_t + y_{t-1} + y_{t-2} + y_{t-3} + y_{t-4})/5$.
(d) FRED (Federal Reserve Bank of St. Louise economic database).
(e) A lscatter plot of y agaisnt x along with a regression line.
(f) $\bar{x} = (8 + 6 + 4 + 2)/4 = 5$ and $\bar{y} = (3 + 5 + 6 + 6)/4 = 5$.

x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$
8	3	3	−2	−6	9
6	5	1	0	0	1
4	6	−1	1	−1	1
2	6	−3	1	−3	9

$b_2 = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) / \sum_{i=1}^n (x_i - \bar{x})^2 = -10/20 = -0.5.$ $\quad b_1 = \bar{y} - b_2\bar{x} = 5 - (-.5) \times 5 = 7.5.$

2.(a) The mean charge increases by $1000b_2 = 1000 \times 1.252196 = 1,252$ dollars.
(b) A 95% confidence interval for β_2 is (.9889, 1.5154) from the regression output.
(c) A 90% confidence interval for β_2 is $b_2 \pm t_{.05,.166} \times s_{b_2}$
$= 1.2522 \pm invttail(166, .05) \times .13333 = 1.2522 \pm 1.654 \times .13333 = 1.2522 \pm .2205 = (1.0317, 1.4727).$
(d) $H_0 : \beta_2 = 0$ against $H_a : \beta_2 \neq 0$.
Using p-value approach: $p = 0.000 < 0.05$. So reject H_0.
Alternatively and equivalently $|t| = 9.39 > t_{.025,166} = invttail(166, .025) = 1.974$. So reject H_0.
Conclude there is a statistically significant relationship at level 0.05.
(e) $H_0 : \beta_2 \leq 0$ against $H_a : \beta_2 > 0$. Important: Claim is the alternative hypothesis.
First $b_2 > 0$ so on the right side for an upper-tail alternative.
Using p-value approach: $p = 0.000/2 = 0.000 < 0.05$. So reject H_0.
Alternatively and equivalently $t = 9.39 > t_{.05,166} = invttail(166, .05) = 1.654$. So reject H_0.
Conclude that agree with claim that mean charge rises with mean cost at level 0.05.
(f) $H_0 : \beta_2 = 1$ against $H_a : \beta_2 \neq 1$.
$t = (b_2 - 1)/s_{b2} = (1.2522 - 1)/.13333 = .2522/.13333 = 1.891.$
Since $|t| = 1.891 > t_{.05,167} = invttail(166, .05) = 1.654$ we reject H_0.
Conclude that mean charge does not rise by \$1 for each \$1 increase in mean cost at level 0.10.

3.(a) $\widehat{y} = b_1 + b_2x = 21432.8 + 1.2522 \times 30000 = \$58,999.$
(b) A wrong answer (no credit) is $1/1.2522 = .7986$.
A full credit answer is that there is not enough information.
(A better answer, this is tricky, is that the slope coefficient from regress y on x is $r_{xy} \times s_y/s_x = \sqrt{.3470} \times 10132/21539 = .2771$.)
(c) This is just the sample mean of `meancharge`: 47509.81.

4.(a) R^2 = RegressionSS / TotalSS = 10/250 = 0.04.
(b) Correlation coefficient $r_{xy} = \sqrt{R^2} = \sqrt{0.04} = 0.2$.
(c) Standard error of the residual = $\sqrt{\text{ResidualSS / (n-2)}} = \sqrt{(250 - 10)/8} = \sqrt{30} = 5.48$.
(d) This produces the residuals $y_i - \widehat{y}_i$.

The course grade will be based on a curve from the combined scores of midterm 1 (22.5%), midterm 2 (22.5%), final (45%) and assignments (10%). **The curve for this exam is only a guide**. Suggested average GPA for this course is 2.4. Curve below has average GPA 2.39 for this exam.

Scores out of	30
75*th* percentile	23 (77%)
Median	19 (63%)
25*th* percentile	15.5 (52%)

A+	27 and above	C+	20 and above
A	25 and above	C	19 and above
A-	24 and above	C-	18 and above
B+	23 and above	D+	17 and above
B	22 and above	D	16 and above
B-	21 and above	D-	14.5 and above