**240A Cameron Winter 2006**
**Department of Economics, U.C.-Davis**

**Midterm Exam: February 8**

Compulsory. Closed book. Worth 32% of course grade.
Read question carefully so you answer the question.
Keep answers as brief as possible.

**Question scores (total 50 points)**

| Question | 1a | 1b | 1c | 1d | 2a | 2b | 2c | 3a | 3b | 3c | 3d | 4a | 4b | 4c | 5a | 5b | 5c | 6a | 6b | 6c | 6d |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Points | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 2 | 4 | 2 | 2 | 2 | 4 | 2 | 2 | 2 | 3 | 3 |

**1.** Given sample values of $(x, y)$ equal to $(0, 0)$, $(2, 4)$ and $(4, 2)$, OLS regression of $y$ on $x$ and intercept gives $\widehat{\beta}_1 = 1.0$ and $\widehat{\beta}_2 = 0.5$. **There is no need to derive these estimates.** Answer the following giving the appropriate algebra (either summation or matrix notation is okay). [If your calculator performs regression do not use this feature. Use first principles.]

**(a)** Show that $s$, the standard error for this regression, equals 2.4495.

**(b)** Calculate $s_{\widehat{\beta}_2}$, the standard error of $\widehat{\beta}_2$. [Use $s = 2.4495$ even if you cannot answer (a)].

**(c)** Calculate a 95 percent confidence interval for $\beta_2$.
[For $t$ with 1, 2 and 3 degrees of freedom, the critical values are 12.706, 4.303 and 3.182.]

**(d)** Calculate $R^2$ for this regression.

**2.** Suppose $\mathbf{b}$ is a $2 \times 1$ vector with entries $b_1$ and $b_2$ and $\mathbf{A}$ is a a $2 \times 2$ matrix with entries $a_{11}$, $a_{12}$, $a_{21}$ and $a_{22}$.

**(a)** Obtain the formula for $\mathbf{b}'\mathbf{A}\mathbf{b}$.

**(b)** Given (a), calculate $\partial\mathbf{b}'\mathbf{A}\mathbf{b}/\partial b_1$ and $\partial\mathbf{b}'\mathbf{A}\mathbf{b}/\partial b_2$.

**(c)** Verify that your answer in (b) implies $\partial\mathbf{b}'\mathbf{A}\mathbf{b}/\partial\mathbf{b} = 2\mathbf{A}\mathbf{b}$ if $\mathbf{A}$ is symmetric..

**3.** Consider the usual multiple regression model with intercept, which can be written as $\mathbf{y} = \beta_1\mathbf{l} + \mathbf{X}_2\boldsymbol{\beta}_2 + \mathbf{u}$, where $\mathbf{l}$ is an $N \times 1$ vector of ones, $\mathbf{X}_2$ is $N \times (k-1)$. Assume $\mathbf{u} \sim \mathcal{N}[\mathbf{0}, \sigma^2\mathbf{I}]$. By subtracting means of regressors, an equivalent transformed model is

$$\mathbf{y} = \alpha_1\mathbf{l} + \mathbf{X}_2^*\boldsymbol{\beta}_2 + \mathbf{u}, \text{ where } \alpha_1 = \beta_1 + \bar{\mathbf{x}}_2'\boldsymbol{\beta}_2,$$

where $\mathbf{X}_2^* = \mathbf{X}_2 - \mathbf{l}\bar{\mathbf{x}}_2'$, with $\bar{\mathbf{x}}_2' = \frac{1}{N}\mathbf{l}'\mathbf{X}_2$ a $1 \times (k-1)$ vector of means of the regressors.

**(a)** Write this transformed model as

$$\mathbf{y} = \mathbf{Z}\boldsymbol{\gamma} + \mathbf{u},$$

for appropriate $\mathbf{Z}$ and $\boldsymbol{\gamma}$.

**(b)** Show that $\mathbf{X}_2^{*\prime}\mathbf{l} = \mathbf{0}$.

**(c)** Use $\widehat{\boldsymbol{\gamma}} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{y}$ and part (b), even if you could not prove it, to show that OLS regression in the transformed model yields, after some algebra,

$$\begin{bmatrix} \widehat{\alpha}_1 \\ \widehat{\boldsymbol{\beta}}_2 \end{bmatrix} = \begin{bmatrix} \bar{y} \\ (\mathbf{X}_2^{*\prime}\mathbf{X}_2^*)^{-1}\mathbf{X}_2^{*\prime}\mathbf{y}. \end{bmatrix}$$

**(d)** And show that the usual OLS estimate of the variance matrix simplifies to

$$\widehat{\mathrm{V}}\begin{bmatrix} \widehat{\alpha}_1 \\ \widehat{\boldsymbol{\beta}}_2 \end{bmatrix} = s^2 \begin{bmatrix} \frac{1}{N} & \mathbf{0} \\ \mathbf{0} & (\mathbf{X}_2^{*\prime}\mathbf{X}_2^*)^{-1} \end{bmatrix}.$$

**4.** Now consider prediction of $\mathrm{E}[y|\mathbf{x}_2 = \mathbf{x}_{2f}]$ using the transformed model of question 3, where you can use the results in 3(c) and 3(d). We use predicted value

$$\widehat{y}_f = \widehat{\alpha}_1 + \mathbf{x}_{2f}^{*\prime}\widehat{\boldsymbol{\beta}}_2,$$

where $\mathbf{x}_{2f}^*$ is a $(k-1)\times 1$ vector with $j^{th}$ entry $(x_{jf} - \bar{x}_j)$.

**(a)** Obtain $\widehat{\mathrm{V}}[\widehat{y}_f]$. [Hint: first write $\widehat{y}_f$ in the form $\mathbf{c}'\widehat{\boldsymbol{\gamma}}$].

**(b)** Hence obtain a 95 percent confidence interval for $\widehat{y}_f$ as a prediction of $\mathrm{E}[y|\mathbf{x}_2 = \mathbf{x}_{2f}]$.

**(c)** For what value of $\mathbf{x}_{2f}$ is this confidence interval the narrowest? Explain your answer.

**5.** Consider the OLS estimator $\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ in the multiple regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u},$$

where $\mathbf{y}$ is an $N \times 1$ vector, $\mathbf{X}$ is an $N \times k$ matrix, $\boldsymbol{\beta}$ is a $k \times 1$ vector and $\mathbf{u}$ is an $N \times 1$ vector. We assume that $\mathbf{X}$ is fixed and $\mathbf{u} \sim N[\mathbf{0},\ \boldsymbol{\Sigma}]$ where $\boldsymbol{\Sigma} \neq \sigma^2\mathbf{I}$.

**(a)** Obtain $\mathrm{E}[\widehat{\boldsymbol{\beta}}]$.

**(b)** Obtain $\mathrm{V}[\widehat{\boldsymbol{\beta}}]$.

**(c)** Give the distribution of $\widehat{\boldsymbol{\beta}}$.

**6.** Consider the attached Stata output that uses data from the study by Joni Hersch, "Compensating Wage Differentials for Gender-Specific Job Injury Risks," American Economic Review, 1998, pp. 598-607. The original data are from the 1994 Current Population Survey, for a sample of males age 18-65.

**Dependent Variable:**

lnwage = Natural logarithm of hourly wage rate in dollars

**Regressors:**

jirate = job injury rate per 100 workers per year for the industry that the individual works in
exper = years of work experience
expersq = experience-squared
educ = years of schooling
union = 1 if union member and 0 otherwise
nonwhite = 1 if nonwhite and 0 otherwise

**(a)** Which regressors are statistically significant at level 0.05 using a two-sided test? Explain.

**(b)** Are the regressors jointly statistically significant at level 0.05? Explain.

**(c)** State how you would set up a test of the null hypothesis that exper and expersq are jointly statistically significant at level 0.05. [There is not enough information to complete the test].

**(d)** Test at level 0.05 the claim that one more year of education is associated with a 10 percent rise in wage. [Note: $\mathrm{d}\ln y/\mathrm{d}x = (\mathrm{d}y/y)/\mathrm{d}x$.] State clearly details of the test and your conclusion.

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. summarize lnwage jirate exper expersq educ union nonwhite

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      lnwage |       182    2.476015    .4783792    1.15268   4.454347
      jirate |       182     4.23074    1.529187   1.195951   7.602587
       exper |       182    20.38736    11.73709          0         55
     expersq |       182     552.647    571.8225          0       3025
        educ |       182    12.21703    2.704791          1         17
-------------+--------------------------------------------------------
       union |       182    .1868132    .3908367          0          1
    nonwhite |       182    .0769231    .2672044          0          1
.
. regress lnwage jirate exper expersq educ union nonwhite

      Source |       SS       df       MS              Number of obs =     182
-------------+------------------------------           F(  6,   175) =   13.71
       Model |  13.2466557     6  2.20777595           Prob > F      =  0.0000
    Residual |   28.174595   175  .160997685           R-squared     =  0.3198
-------------+------------------------------           Adj R-squared =  0.2965
       Total |  41.4212507   181  .228846689           Root MSE      =  .40125

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      jirate |  -.0173072   .0198123    -0.87   0.384    -.0564089    .0217945
       exper |   .0250062    .009082     2.75   0.007     .0070818    .0429307
     expersq |  -.0002364   .0001918    -1.23   0.219    -.0006149    .0001421
        educ |   .0836888   .0124466     6.72   0.000     .0591241    .1082536
       union |   .1123168   .0778764     1.44   0.151    -.0413811    .2660147
    nonwhite |  -.2754669   .1119563    -2.46   0.015    -.4964253   -.0545085
       _cons |   1.147848   .2028857     5.66   0.000       .74743    1.548265
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