240D Cameron Fall 2005
Department of Economics, U.C.–Davis

Final Exam: December 16 2005

Compulsory. Closed book. 2 hours. Worth 50% of course grade.
Read question carefully so you answer the question.

Question scores (total 50 points and 50% of course grade)

Question	1	2	3	4	5
Points	12	12	10	6	10

1. Estimation.
Consider the estimator $\widehat{\boldsymbol{\beta}}$ that minimizes

$$Q_N(\boldsymbol{\beta}) = \left(\frac{1}{N}\sum_{i=1}^{N}\mathbf{z}_i(y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}))\right)'\left(\frac{1}{N}\sum_{i=1}^{N}\mathbf{z}_i(y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}))\right),$$

where $\mathbf{x}_i$ and $\boldsymbol{\beta}$ are $k \times 1$ vectors and $\mathbf{z}_i$ is an $m \times 1$ vector with $m > k$.

(a) Provide a motivation for this estimator as arising from some underlying model or structure.

(b) Show that the estimator $\widehat{\boldsymbol{\beta}}$ solves

$$\left(\frac{1}{N}\sum_{i=1}^{N}\exp(\mathbf{x}_i'\widehat{\boldsymbol{\beta}})\mathbf{x}_i\mathbf{z}_i'\right)\left(\frac{1}{N}\sum_{i=1}^{N}\mathbf{z}_i(y_i - \exp(\mathbf{x}_i'\widehat{\boldsymbol{\beta}}))\right) = \mathbf{0}.$$

(c) Rewrite the result in part (b) as

$$\mathbf{G}_N(\widehat{\boldsymbol{\beta}})'\sqrt{N}\mathbf{g}_N(\widehat{\boldsymbol{\beta}}) = \mathbf{0}$$

for appropriately defined $\mathbf{G}_N(\widehat{\boldsymbol{\beta}})$ and $\mathbf{g}_N(\widehat{\boldsymbol{\beta}})$.

(d) Then take an exact first-order Taylor series expansion of $\mathbf{g}_N(\widehat{\boldsymbol{\beta}})$ about $\boldsymbol{\beta}_0$.

(e) Substitute this expression back into (c) and solve to get the limit distribution for this estimator, stating clearly any assumptions made.
You can give your answer in general form given (c), there is no need to specialize to this particular question.

(f) Given your assumptions state, with explanation, whether or not a more efficient estimator than $\widehat{\boldsymbol{\beta}}$ defined at the start can be obtained. If more efficient estimation is possible, then state how to do it (give an appropriate objective function).

2. Discrete and multinomial choice.

(a) Derive the first order conditions for the probit model.

(b) A sample has means $\bar{y} = 0.8$ and $\bar{x} = 3$. Probit regression of y on the scalar x leads to an estimated intercept of 0.4 (with standard error 0.1) and slope of 0.2 (with standard error of 0.05). Provide a meaningful numerical interpretation of the marginal effect of a change in x.

(c) Suppose that a person chooses the alternative with highest utility. There are two mutually exclusive alternatives, with utility of alternatives 0 and 1 for individual i given by

$$\begin{aligned} U_{i0} &= \mathbf{x}_i'\boldsymbol{\beta}_0 + \varepsilon_{i0} \\ U_{i1} &= \mathbf{x}_i'\boldsymbol{\beta}_1 + \varepsilon_{i1}, \end{aligned}$$

where ε_{i0} and ε_{i1} are independent standard normal distributed. We observe $y_i = 1$ if alternative 1 is chosen and $y_i = 0$ otherwise. Obtain the probability that $y_i = 1$.

(d) Provide, with brief derivation, the log-likelihood function for the ordered logit model with three alternatives, and state clearly which parameters can be estimated.

3. Censoring, sample selection and truncation.

Throughout this question we suppress the subscript i for the individual. Consider the latent variable model

$$y = \mathbf{x}'\boldsymbol{\beta} + \alpha d + \varepsilon,$$

where $\mathbf{x}$ are exogenous regressors and d is an indicator variable for whether a person participates in a training program. The complication is that d is endogenous. Specifically, we observe

$$d = \begin{cases} 1 & \text{if } d^* > 0 \\ 0 & \text{if } d^* \leq 0, \end{cases}$$

where

$$d^* = \mathbf{w}'\boldsymbol{\gamma} + v.$$

We suppose that

$$\varepsilon = \delta v + u,$$

where $v \sim \mathcal{N}[0,1]$ and $u \sim \mathcal{N}[0,\sigma_u^2]$ and u and v are independent.

(a) Show that

$$\mathrm{E}[y|d=1,\mathbf{x},\mathbf{w}] = \mathbf{x}'\boldsymbol{\beta} + \alpha + \mathrm{E}[\varepsilon|v > -\mathbf{w}'\boldsymbol{\gamma}].$$

(b) Using part (a), even if you could not derive it, and the result that

$$\mathrm{E}[z|z > -c] = \phi(c)/\Phi(c) \quad \text{for} \quad z \sim \mathcal{N}[0,1],$$

obtain an expression for $\mathrm{E}[\varepsilon|v > -\mathbf{w}'\boldsymbol{\gamma}]$ and hence for $\mathrm{E}[y|d=1,\mathbf{x},\mathbf{w}]$.

(c) Given your answer in (b), state how you would obtain consistent estimates of $\boldsymbol{\beta}$ given access to probit and OLS programs.

(d) Is it also possible to estimate α using only the expression for $\mathrm{E}[y|d=1,\mathbf{x},\mathbf{w}]$ obtained in part (b)? Give a brief explanation.

4. Estimation of a nonlinear regression model led to the estimates

$$y = \exp(\underset{(0.232)}{-3.192} + \underset{(1.741)}{5.664}x),$$

where standard errors are given in parentheses.
The model was then bootstrapped. Let $\widehat{\beta}_b^*$ denote the estimated slope coefficient from the b^{th} bootstrap replication, and define $t_b^* = (\widehat{\beta}_b^* - 5.664)/s_{\widehat{\beta}_b^*}$. The results for 999 bootstrap replications were:

	$\widehat{\beta}_b^*$	t_b^*
Mean	5.716	0.026
St.Dev.	1.939	1.047
Median	5.772	0.062
2.5 *percentile*	1.501	−2.183
97.5 *percentile*	9.484	2.066

(a) Test $H_0 : \beta = 0$ against $H_a : \beta \neq 0$ at level 0.05 using the usual asymptotic Wald test.

(b) Obtain the bootstrap estimate of the standard error of $\widehat{\beta}$ and use this estimate to test $H_0 : \beta = 0$ against $H_a : \beta \neq 0$ at level 0.05.

(c) Test $H_0 : \beta = 0$ against $H_a : \beta \neq 0$ at level 0.05 using a bootstrap with asymptotic refinement.

5. Consider the linear panel data model

$$y_{it} = \alpha_i + \mathbf{x}_{it}'\boldsymbol{\beta} + \varepsilon_{it}, \quad i = 1, ..., N, \ t = 1, ..., T,$$

where $\boldsymbol{\beta}$ are parameters to be estimated, α_i, $i = 1, ..., N$ are individual specific effects, ε_{it} are zero mean errors independent over i, and $N \to \infty$ while T is small.

(a) Use an appropriate differencing transformation applied to the y_{it} equation that leads to the within estimator and give the formula for the within estimator.

(b) State how to stack the transformed model in part (a) as

$$\widetilde{\mathbf{y}}_i = \widetilde{\mathbf{X}}_i\boldsymbol{\beta} + \widetilde{\boldsymbol{\varepsilon}}_i,$$

(c) Hence state how to consistently estimate the variance matrix of the within estimator under the assumptions that $\text{Cov}[\varepsilon_{it}, \varepsilon_{is}] \neq 0$, and for $i = j$, $\text{Cov}[\varepsilon_{it}, \varepsilon_{js}] = 0$.

(d) Suppose that $\alpha_i \sim$ iid $[0, \sigma_\alpha^2]$ and that $\varepsilon_{it} \sim$ iid $[0, \sigma_\varepsilon^2]$. Is the within estimator optimal (i.e. consistent and fully efficient?) Give a brief explanation.

(e) Suppose that α_i are correlated with x_{it} and that $\varepsilon_{it} \sim$ iid $[0, \sigma_\varepsilon^2]$. Is the within estimator optimal (i.e. consistent and fully efficient?)? Give a brief explanation.