240D Cameron Fall 2008
Department of Economics, U.C.–Davis

Final Exam: December 11 2008

Compulsory. Closed book. 2 hours. Worth 50% of course grade.
Read question carefully so you answer the question.

Question scores (total 50 points and 50% of course grade)

Question	1	2	3	4	5
Points	12	12	10	6	10

1. Estimation.
Consider the binary dependent variable y that takes value 1 with probability $\Lambda(\mathbf{x}_i'\boldsymbol{\beta})$ and value 0 with probability $1-\ \Lambda(\mathbf{x}_i'\boldsymbol{\beta})$, where $\Lambda(z) = e^z/(1+e^z)$, $\boldsymbol{\beta}$ is an unknown $K \times 1$ parameter vector and $\mathbf{x}_i$ is a stochastic $K \times 1$ vector.

(a) Give the log-likelihood function for $\boldsymbol{\beta}$ and the associated first-order conditions for the MLE for $\boldsymbol{\beta}$.

(b) Obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta})$ by taking a first-order Taylor series expansion of the first-order conditions. Your derivation can be brief (there is no need to justify central limit theorems and laws of large numbers).

(c) Suppose that there are regressors that are endogenous, but there are $\mathbf{z}$ instruments available where the number of instruments exceed the number of regressors. Write down the objective function that provides a consistent estimator of $\boldsymbol{\beta}$, and state the essential condition(s) needed for consistent estimation.

(d) Now specialize to the just-identified case. Give the formula for the asymptotic variance matrix of the estimator given in part (c).

2. Discrete and multinomial choice.

(a) A sample has means $\bar{y} = 0.8$ and $\bar{x} = 3$. Logit regression of y on the scalar x leads to an estimated intercept of 0.4 (with standard error 0.1) and slope of 0.2 (with standard error of 0.05). Provide a meaningful numerical interpretation of the marginal effect of a change in x.

(b) Derive the probit model based on observing $y_i = 1$ if $y_i^* = \mathbf{x}_i'\boldsymbol{\beta} + u_i > 0$, where u_i is normally distributed.

(c) Consider estimation of a three-outcome model with $\Pr[y_i = j] = F_j(\mathbf{x}_i'\boldsymbol{\beta}_j)$, $j = 1, ..., 3$. Give the log-likelihood for the model.

(d) Derive the probabilities for a three-outcome ordered probit model.

(e) State the probabilities for a three-outcome multinomial logit model.

(f) Present the additive random utility model in the case of three alternative model and state what assumptions on this model yield your answer in part (e).

3. Censoring, sample selection and truncation.
Consider a Poisson regression model where y^* has density $f^*(y^*) = e^{-\mu}\mu^y/y^*!$, $y^* = 0, 1, 2, ...$ and given regressors $\mathbf{x}$, $\mu_i = \mathrm{E}[y_i|\mathbf{x}_i] = \exp(\mathbf{x}_i'\boldsymbol{\beta})$. Data are independent over i.
Suppose we only observe $y = y^*$ when $y^* \geq 1$. When $y^* = 0$ we do not observe anything (so the data are truncated).

(a) Obtain the density $f(y|\mathbf{x})$ of the observed y.

(b) Hence obtain the first-order conditions for the MLE of $\boldsymbol{\beta}$.

(c) Show that $\mathrm{E}[y] = \mathrm{E}[y^*|y^* \geq 1] = \mu/(1 - e^{-\mu})$. [There is some algebra here.]

(d) Using the result in part (c), even if you could not derive it, propose a consistent estimator for $\boldsymbol{\beta}$ other than the MLE.

(e) This part is unrelated to parts (a)-(d).
Present the underlying equations for Heckman's sample selection model and state how to estimate the regression parameters of this model using Heckman's two-step estimator. There is no need for any proofs, just give the results.

4. Testing.
Consider the following Stata commands and output.

```
. poisson docvis chronic, vce(robust) nolog
Poisson regression                                Number of obs   =         50
                                                  Wald chi2(1)    =       3.64
                                                  Prob > chi2     =     0.0565
Log pseudolikelihood = -238.75384                 Pseudo R2       =     0.0917
      docvis |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     chronic |   .9833014
       _cons |   1.031602
------------------------------------------------------------------------------
. global beta = _b[chronic]
. bootstrap bchronic=_b[chronic] sechronic=_se[chronic], reps(999)     ///
>   saving(bootoutput, replace) seed(10101) nodots:                    ///
>   poisson docvis chronic, vce(robust)
. use bootoutput, clear
. generate tstar = (bchronic-$beta)/sechronic
. centile tstar, centile(2.5 97.5)
                                                    -- Binom. Interp. --
    Variable |     Obs  Percentile     Centile        [95% Conf. Interval]
-------------+-------------------------------------------------------------
       tstar |     999        2.5    -2.756196       -3.030972   -2.567785
             |               97.5     2.568691          2.3092    2.917802
. summarize
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
    bchronic |       999    .9588541    .5404076  -.6184664   2.698503
   sechronic |       999    .4613141    .0799398   .2429544   .7358097
       tstar |       999   -.0874198      1.3004  -4.435354   4.611353
```

(a) Perform a likelihood ratio test at level 0.05 of $H_0 : \beta_{chronic} = 0$ against $H_a : \beta_{chronic} \neq 0$. State what you conclude.

(b) Perform a Wald test at level 0.05 of $H_0 : \beta_{chronic} = 0$ against $H_a : \beta_{chronic} \neq 0$ without asymptotic refinement. State what you conclude.

(c) Perform a Wald test at level 0.05 of $H_0 : \beta_{chronic} = 0$ against $H_a : \beta_{chronic} \neq 0$ with asymptotic refinement. State what you conclude.

5. Consider the linear panel data model

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}, \quad i = 1, ..., N, \; t = 1, ..., T,$$

where $\boldsymbol{\beta}$ are parameters to be estimated, α_i, $i = 1, ..., N$ are individual specific effects, ε_{it} are iid $[0, \sigma_\varepsilon^2]$ errors and $N \to \infty$ while T is small.

(a) Use an appropriate differencing transformation applied to the y_{it} equation that leads to the within estimator and give the formula for the within estimator.

(b) Explain the Hausman test for fixed versus random effects.
State clearly the null and alternative hypotheses, the test statistic and the critical region.

(c) How, if at all, will your answer in (b) will change if ε_{it} is not i.i.d $(0, \sigma_\varepsilon^2)$. Give details.

(d) Stack the regression model appropriately and show that premultiplication by the matrix $\mathbf{Q} = \mathbf{I}_T - \frac{1}{T}\mathbf{e}\mathbf{e}'$ leads to elimination of α_i.