240D Cameron Fall 2011
Department of Economics, U.C.–Davis
Final Exam: March 16 2011

Compulsory. Closed book. 2 hours. Worth 50% of course grade.
Read question carefully so you answer the question.

Question scores (total 50 points and 50% of course grade)

Question	1	2	3	4	5
Points	10	10	8	12	10

1. Consider estimation of a model with density

$$f(y_i) = e^{-(y_i - \mathbf{x}_i'\boldsymbol{\beta})} \exp(-e^{-(y_i - \mathbf{x}_i'\boldsymbol{\beta})}),$$

where $-\infty < y < \infty$ and $\mathrm{E}[y_i|\mathbf{x}_i] = c + \mathbf{x}_i'\boldsymbol{\beta}$ where $c \simeq 0.57722$ is Euler's constant, $\mathbf{x}$ includes an intercept, data are independent over i, and in the d.g.p. $\boldsymbol{\beta} = \boldsymbol{\beta}_0$.
In the following your derivations can be as brief as possible.

(a) Obtain the first-order equations for the ML estimator $\widehat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$.

(b) Give the limit distribution for $\widehat{\boldsymbol{\beta}}$, assuming the density is correctly specified.

(c) Do you think that $\widehat{\boldsymbol{\beta}}$ remains consistent if $\mathrm{E}[y_i|\mathbf{x}_i] = c + \mathbf{x}_i'\boldsymbol{\beta}$, but other aspects of the density are misspecified? Explain your answer.

(d) Present an alternative method of moments estimator of $\boldsymbol{\beta}$.

(e) Give the limit distribution for your estimator in (d).

2.(a) Provide a formal definition of convergence in probability.

(b) Provide a formal definition of convergence in distribution.

(c) Suppose $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0) \xrightarrow{d} \mathcal{N}[0,\ \mathrm{V}_0]$. Obtain the limit distribution of $\mathbf{g}(\widehat{\boldsymbol{\beta}})$ where $\mathbf{g}(\cdot)$ is a continuous function.

(d) Give the formulae for the MEM and AME in the probit model.

(e) Give the commands to compute these in either Stata 10 or Stata 11.

3. Discrete, multinomial and selection.
Consider a linear regression model based on the latent variable

$$y_i^* = \mathbf{x}_i'\boldsymbol{\beta} + u_i,$$

where the errors u_i are logistic distributed with density and c.d.f. given by

$$f(u) = \frac{e^{-u}}{(1+e^{-u})^2} \qquad \text{and} \qquad F(u) = \frac{1}{1+e^{-u}}.$$

Note that the density is symmetric, so $f(u) = f(-u)$. For the logistic

$$\mathrm{E}[u|u \leq c] = c + \tfrac{\ln(1-F(c))}{F(c)}.$$

In this question the variable y_i^* is not completely observed.

(a) Suppose we observe only

$$\begin{aligned} y_i &= 1 \quad \text{if } y_i^* \geq 0 \\ y_i &= 0 \quad \text{if } y_i^* < 0 \end{aligned}$$

Give with justification the objective function for a consistent estimator of $\boldsymbol{\beta}$.

(b) Suppose we observe only

$$\begin{aligned} y_i &= 2 \quad \text{if } y_i^* \geq \alpha \\ y_i &= 1 \quad \text{if } 0 \leq y_i^* < \alpha \\ y_i &= 0 \quad \text{if } y_i^* < 0. \end{aligned}$$

State a method to consistently estimate $\boldsymbol{\beta}$ and α. For this part you may provide little detail.

(c) Suppose we observe only

$$y_i = y_i^* \quad \text{if } y_i^* \geq 0$$

Give with justification the objective function for the MLE of $\boldsymbol{\beta}$.

(d) In the same situation as in part (c), give an alternative consistent estimator for $\boldsymbol{\beta}$ that uses nonlinear least squares.

4.(a) Provide complete Stata code that demonstrates that the Stata Tobit command leads to a consistent estimator in the Tobit model.

(b) Present the Heckman two-step estimator for the sample selection model. A brief answer will do.

(c) State how you could obtain valid standard errors for your estimator in part (b) using the bootstrap, assuming that data are i.i.d. over i.

(d) Suppose that if $\mathbf{x}_i$ is exogenous, then $\mathrm{E}[y_i|\mathbf{x}_i] = \Lambda(\mathbf{x}_i'\boldsymbol{\beta})$ where $\Lambda(\cdot)$ is the logistic c.d.f. Instead, some of the components of $\mathbf{x}_i$ are endogenous, but there exist many available instruments $\mathbf{z}_i$, with $\dim[\mathbf{z}_i] > \dim[\mathbf{x}_i]$. Give the key moment condition and the corresponding objective function for a GMM estimator of $\boldsymbol{\beta}$.

(e) For your estimator in part (d), give the best choice of weighting matrix, assuming independence over i. And provide a way to consistently estimate this weighting matrix.

5. Keep answers brief.

(a) Give the equation defining the fixed effects models and state three different methods for computing the FE (or within) estimator.

(b) Under what assumptions is the random effects estimator consistent and fully efficient?

(c) Outline the usual Hausman formulation for the Hausman test for the presence of fixed effects and comment on any weaknesses of this test.

(d) Consider pooled OLS estimation if y_{it} on $\mathbf{x}_{it}$ where for this part of the question the intercept is included in $\mathbf{x}_{it}$. Assume errors are independent over i but correlated over t for given i. Provide the formula for the cluster-robust estimate of the variance-covariance matrix of the OLS estimator. [Hint: Stack all observations over t for given i and then proceed.]