240D Cameron Fall 2004
Department of Economics, U.C.-Davis

Midterm Exam: November 3

Compulsory. Closed book. Worth 35% of course grade.
Read question carefully so you answer the question.
Keep answers as brief as possible.

Question scores (total 35 points)

Question	1a	1b	1c	1d	2a	2b	2c	2d	3a	3b	4	5a	5b
Points	2	4	2	2	3	3	2	2	2	3	5	3	2

1. Consider the estimator $\widehat{\boldsymbol{\beta}}$ that minimizes

$$Q_N(\boldsymbol{\beta}) = \frac{1}{N}\sum_{i=1}^{N}(y_i - \mathbf{x}_i{}'\boldsymbol{\beta})^4,$$

where y is a scalar, $\mathbf{x}$ is a nonstochastic column vector and $\boldsymbol{\beta}$ is a $K \times 1$ column vector. **Note that we have used the fourth power here.**
In the true model (the dgp)

$$y_i = \mathbf{x}_i{}'\boldsymbol{\beta}_0 + u_i,$$

where u_i is normally distributed with mean 0 and variance σ_0^2. Note that that the odd moments of u_i equal zero and for even moments $\mathrm{E}[u_i^{2k}] = \sigma_0^{2k}(2k)!/(2^k k!)$ giving $\mathrm{E}[u_i^2] = \sigma^2$, $\mathrm{E}[u_i^4] = 3\sigma^4$, $\mathrm{E}[u_i^6] = 15\sigma^6$, $\mathrm{E}[u_i^8] = 105\sigma^8$...

If you need to make other assumptions, state them as you go along.
You can apply laws of large numbers and central limit theorems without formally verifying necessary conditions for their use (except 1(d) requires further detail).
You need not verify any second-order conditions.

(a) Show that $Q_N(\boldsymbol{\beta})$ can be rewritten as

$$\begin{aligned} Q_N(\boldsymbol{\beta}) &= \frac{1}{N}\sum_{i=1}^{N} u_i^4 + \frac{1}{N}\sum_{i=1}^{N} 4u_i^3\mathbf{x}_i'(\boldsymbol{\beta}_0 - \boldsymbol{\beta}) + \frac{1}{N}\sum_{i=1}^{N} 6u_i^2[\mathbf{x}_i'(\boldsymbol{\beta}_0 - \boldsymbol{\beta})]^2 \\ &\quad + \frac{1}{N}\sum_{i=1}^{N} 4u_i[\mathbf{x}_i'(\boldsymbol{\beta}_0 - \boldsymbol{\beta})]^3 + \frac{1}{N}\sum_{i=1}^{N}[\mathbf{x}_i'(\boldsymbol{\beta}_0 - \boldsymbol{\beta})]^4 \end{aligned}$$

[Hint: $y_i - \mathbf{x}_i{}'\boldsymbol{\beta} = y_i - \mathbf{x}_i{}'\boldsymbol{\beta}_0 + \mathbf{x}_i{}'\boldsymbol{\beta}_0 - \mathbf{x}_i{}'\boldsymbol{\beta}$ and then use definition of u_i].

(b) Obtain $Q_0(\boldsymbol{\beta}) = \operatorname{plim} Q_N(\boldsymbol{\beta})$.
[Note: Start with $Q_N(\boldsymbol{\beta})$ given in part (a) even if you could not derive part (a)].

(c) Using (b) prove that $\widehat{\boldsymbol{\beta}}$ is consistent for $\boldsymbol{\beta}_0$.

(d) Formally obtain the probability limit of $N^{-1}\sum_i 4u_i[\mathbf{x}_i'(\boldsymbol{\beta}_0 - \boldsymbol{\beta})]^3$ by applying a law of large numbers. State any assumptions that are needed.

2. Continue with the same model and estimator as question 1.

(a) Obtain the limit distribution of $\sqrt{N}\frac{\partial Q_N}{\partial \beta}\Big|_{\boldsymbol{\beta}_0}$.

(b) Obtain the probability limit of $\frac{\partial^2 Q_N}{\partial \beta \partial \beta'}\Big|_{\boldsymbol{\beta}_0}$.

(c) Combine the above to obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.

(d) Give a method to obtain an estimate of the asymptotic variance matrix of $\widehat{\boldsymbol{\beta}}$.

3. Continue with the same model and estimator as question 1, except that here we change slightly the dgp.

(a) Suppose the dgp in question 1 is changed so that u_i is not normally distributed (so e.g. $\mathrm{E}[u^4] \neq 3\sigma^4$) though it is still iid and symmetrically distributed.
Will $\widehat{\boldsymbol{\beta}}$ remain consistent?
[Hint: How will your answers in question 1 parts (b) and (c) change?]

(b) Suppose the dgp in question 1 is changed so that u_i is no longer normal.
Leaving aside issues of consistency, state how to obtain a White-type robust estimate of the variance covariance matrix of the estimator.
[Hint: What effects will this have on question 2 parts (a) and (b)?]

4. Give a way to obtain a numerical estimate of $\widehat{\boldsymbol{\beta}}$ in question 1. Your answer should include (1) the name of the iterative method; (2) the appropriate mathematics; and (3) a reasonable starting value for $\boldsymbol{\beta}$ in the iterations.

5. Discrete choice models.
Consider the logit model with

$$\begin{aligned} \Pr[y_i &= 1|\mathbf{x}_i] = \Lambda(\mathbf{x}_i'\boldsymbol{\beta}) \\ \Pr[y_i &= 0|\mathbf{x}_i] = 1 - \Lambda(\mathbf{x}_i'\boldsymbol{\beta}), \end{aligned}$$

where $\Lambda(z) = e^z/(1+e^z)$

(a) Give the density and hence the log-likelihood for the logit model.

(b) For this model, give a formula for the marginal effect of changes in the regressors on the conditional probability that $y = 1$.