240D Cameron Fall 2005
Department of Economics, U.C.-Davis

Midterm Exam: November 2

Compulsory. Closed book. Worth 35% of course grade.
Read question carefully so you answer the question.
Keep answers as brief as possible.

Question scores (total 35 points)

Question	1a	1b	1c	1d	2a	2b	2c	2d	2e	3	5a	5b	5c	5d	5e
Points	2	4	2	2	1	1	4	2	2	5	2	2	2	2	2

1. Consider a positive-valued dependent variable y that has density

$$f(y) = 2\gamma^{-2} y \exp\left(-\left(\frac{y}{\gamma}\right)^2\right), \quad y > 0, \quad \gamma > 0.$$

For this distribution it can be shown that $\mathrm{E}[y] = \gamma\Gamma(1.5)$, $\mathrm{E}[y^2] = \gamma^2$, $\mathrm{V}[y] = \gamma^2 - \gamma^2(\Gamma(1.5))^2$ where $\Gamma(\cdot)$ is the gamma function and $\Gamma(1.5) \simeq 0.8862$. Also $\mathrm{E}[y^4] = 2\gamma^4$ and $\mathrm{E}[\ln y] = \ln\gamma - c/2$ where $c \simeq 0.5772$ is Euler's constant.

Here we introduce regressors and suppose that the parameter γ depends on regressors according to

$$\gamma_i = \exp(\mathbf{x}_i'\boldsymbol{\beta}),$$

where $\boldsymbol{\beta}$ is an unknown $k \times 1$ parameter vector and $\mathbf{x}_i$ is a nonstochastic $k \times 1$ vector.
Thus $\mathrm{E}[y_i|\mathbf{x}_i] = \exp(\mathbf{x}_i'\boldsymbol{\beta}) \times \Gamma(1.5)$ and so on.
In the dgp $(y_i, \mathbf{x}_i)$ are independent over i with the preceding density for $y_i|\mathbf{x}_i$ and $\boldsymbol{\beta} = \boldsymbol{\beta}_0$.

If you need to make other assumptions, state them as you go along.
You can apply laws of large numbers and central limit theorems without formally verifying necessary conditions for their use (except 1(d) requires further detail).
You need not verify any second-order conditions.

(a) Show that the MLE for $\boldsymbol{\beta}$ maximizes

$$Q_N(\boldsymbol{\beta}) = \frac{1}{N}\sum_{i=1}^{N}\left\{\ln 2 - 2\mathbf{x}_i'\boldsymbol{\beta} + \ln y_i - \left(\frac{y_i}{\exp(\mathbf{x}_i'\boldsymbol{\beta})}\right)^2\right\}.$$

(b) Obtain $Q_0(\boldsymbol{\beta}) = \mathrm{plim}\, Q_N(\boldsymbol{\beta})$.

(c) Using (b) prove that the local max to $Q_N(\boldsymbol{\beta})$ is consistent for $\boldsymbol{\beta}_0$. State any assumptions made.

(d) Now state what LLN you would use to verify part (b) and what additional information, if any, is needed to apply this law. A brief answer will do. There is no need for a formal proof.

2.(a) Show that the derivative with respect to $\boldsymbol{\beta}$ can be written as

$$\frac{\partial Q_N}{\partial \boldsymbol{\beta}} = \frac{1}{N}\sum_{i=1}^{N} 2 \times \frac{y_i^2 - (\exp(\mathbf{x}_i'\boldsymbol{\beta}))^2}{g(\mathbf{x}_i'\boldsymbol{\beta})}\mathbf{x}_i,$$

for some function $g(\mathbf{x}_i'\boldsymbol{\beta})$, and give the function $g(\cdot)$.

(b) What essential condition do the first-order conditions indicate needs to be satisfied for $\widehat{\boldsymbol{\beta}}$ to be consistent?

(c) Give the limit distribution of $\sqrt{N}\frac{\partial Q_N}{\partial \boldsymbol{\beta}}\Big|_{\boldsymbol{\beta}_0}$.
Hint: $\mathrm{V}[y^2] = \mathrm{E}[(y^2)^2] - (\mathrm{E}[y^2])^2 = \mathrm{E}[y^4] - (\mathrm{E}[y^2])^2 = 2\gamma^4 - (\gamma^2)^2 = \gamma^4$.

(d) Obtain the probability limit of $\frac{\partial^2 Q_N}{\partial \boldsymbol{\beta}\partial \boldsymbol{\beta}'}\Big|_{\boldsymbol{\beta}_0}$.

(e) Combine the above to obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.

3. Give a way to obtain a numerical estimate of $\widehat{\boldsymbol{\beta}}$ in question 1.
Your answer should include
(1) the name of the iterative method;
(2) the appropriate mathematics; and
(3) a way to obtain a reasonable starting value for $\boldsymbol{\beta}$ in the iterations.

4. Logit regression of y on the scalar x leads to an intercept of 0.2 and a slope coefficient of 1.6. The corresponding standard errors are 0.8 and 0.4. For the data set used $\bar{y} = 0.75$, $\bar{x} = 0.5$ and $N = 100$. Note that $\Lambda(\mathbf{x}'\boldsymbol{\beta}) = \exp(\mathbf{x}'\boldsymbol{\beta})/[1 + \exp(\mathbf{x}'\boldsymbol{\beta})]$.

(a) Test at level 0.05 the hypothesis that the slope coefficient equals 1.0.

(b) Give an estimate of the marginal effect of a one unit change in x. Explain your answer.

(c) The predicted probabilities for the logit model are found on average to equal the sample frequency $\bar{y}$. Does this mean that the logit model is a good model for these data? Explain.

(d) Suppose a probit model was estimated with the same data. How, if at all, would you expect the probit estimates to differ in a substantial way from the logit estimates.

(e) Suppose you don't have a logit model package. But you do have a nonlinear least squares package. Would it be possible to estimate the logit model and obtain valid standard errors? Give a brief explanation.