240D Cameron Fall 2008
Department of Economics, U.C.-Davis

Midterm Exam: October 23

Compulsory. Closed book. Worth 35% of course grade.
Read question carefully so you answer the question.
Keep answers as brief as possible.

Question scores (total 35 points)

Question	1a	1b	1c	1d	1e	2a	2b	2c	2d	2e	3a	3b	3c	4a	4b
Points	2	4	2	2	2	2	4	3	2	2	2	2	2	2	2

1. Consider the random variable y with density

$$f(y) = \lambda \exp(-\lambda y), \quad y > 0, \quad \lambda > 0.$$

For this distribution it can be shown that $\mathrm{E}[y] = 1/\lambda$ and $\mathsf{V}[y] = 1/\lambda^2$.

Here we introduce regressors and suppose that in the true model the parameter λ depends on regressors according to

$$\lambda_i = \exp(\mathbf{x}_i'\boldsymbol{\beta}),$$

where $\boldsymbol{\beta}$ is an unknown $K \times 1$ parameter vector and $\mathbf{x}_i$ is a nonstochastic $K \times 1$ vector.
Note that this parameterization implies that $\mathrm{E}[y_i|\mathbf{x}_i] = \exp(-\mathbf{x}_i'\boldsymbol{\beta})$.
In the dgp the regressors $\mathbf{x}_i$ are nonstochastic and $y_i|\mathbf{x}_i$ is independent over i with the preceding density for $y_i|\mathbf{x}_i$ and $\boldsymbol{\beta} = \boldsymbol{\beta}_0$.

If you need to make other assumptions, state them as you go along.
You can apply laws of large numbers and central limit theorems without formally verifying necessary conditions for their use (except 1(d) requires further detail).
You need not verify any second-order conditions.

(a) Show that the MLE for $\boldsymbol{\beta}$ maximizes (upon scaling by N^{-1})

$$Q_N(\boldsymbol{\beta}) = \frac{1}{N}\sum_{i=1}^{N} \left\{\mathbf{x}_i'\boldsymbol{\beta} - y_i \exp(\mathbf{x}_i'\boldsymbol{\beta})\right\}.$$

(b) Obtain $Q_0(\boldsymbol{\beta}) = \operatorname{plim} Q_N(\boldsymbol{\beta})$.

(c) Using (b) prove that the local max to $Q_N(\boldsymbol{\beta})$ is consistent for $\boldsymbol{\beta}_0$. State any assumptions made.

(d) Now state what LLN you would use to verify part (b) and what additional information, if any, is needed to apply this law. A brief answer will do. There is no need for a formal proof.

(e) If the density $f(y)$ is misspecified will the MLE in this example be consistent? Explain.

2.(a) Show that the derivative with respect to $\boldsymbol{\beta}$ can be written as

$$\frac{\partial Q_N}{\partial \boldsymbol{\beta}} = -\frac{1}{N}\sum_i (y_i - \exp(-\mathbf{x}_i'\boldsymbol{\beta}))\exp(\mathbf{x}_i'\boldsymbol{\beta})\mathbf{x}_i.$$

(b) Give the limit distribution of $\sqrt{N}\frac{\partial Q_N}{\partial \boldsymbol{\beta}}\Big|_{\boldsymbol{\beta}_0}$.

(c) Obtain the probability limit of $\frac{\partial^2 Q_N}{\partial \boldsymbol{\beta}\partial \boldsymbol{\beta}'}\Big|_{\boldsymbol{\beta}_0}$.

(d) Combine the above to obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.

(e) Hence provide a test of $H_0 : \beta_j = 0$ against $H_a : \beta_j \neq 0$ at level 0.05.

3.(a) Give an example of a central limit theorem in the scalar case, stating clearly the assumptions of the central limit theorem and the conclusion.

(b) Give the asymptotic distribution of the nonlinear least squares estimator when errors are heteroskedastic. There is no need to give a derivation just give the results. Provide a formula for the estimate of the variance matrix of the estimator.
[If you can't answer this for NLS then for reduced credit answer this for OLS].

(c) For general estimator $\widehat{\boldsymbol{\theta}}$ that maximizes $Q(\boldsymbol{\theta})$ with first order-condition $g(\widehat{\boldsymbol{\theta}}) = \mathbf{0}$ give the formula for an iterative method to calculate $\widehat{\boldsymbol{\theta}}$.

4. Poisson regression yields the following estimates

```
Poisson regression                              Number of obs   =       4412
                                                Wald chi2(4)    =     594.72
                                                Prob > chi2     =     0.0000
Log pseudolikelihood = -18503.549               Pseudo R2       =     0.1930

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             |               Robust
      docvis |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     private |   .7986652   .1090014     7.33   0.000     .5850263    1.012304
     chronic |   1.091865   .0559951    19.50   0.000     .9821167    1.201614
      female |   .4925481   .0585365     8.41   0.000     .3778187    .6072774
      income |    .003557   .0010825     3.29   0.001     .0014354    .0056787
       _cons |  -.2297262   .1108732    -2.07   0.038    -.4470338   -.0124186
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The dependent variable is the annual number of doctor visits. The first three regressors are indicator variables (equal to 1 if, respectively, have private insurance, have a chronic condition and are female). The fourth regressor is annual income in thousands of dollars.

(a) Provide an interpretation of the estimated coefficient of variable `income`.

(b) Calculate the marginal effect of a $1,000 increase in annual income for a privately insured woman who has no chronic conditions and has annual income of $10,000.