240D Cameron Winter 2011
Department of Economics, U.C.-Davis

Midterm Exam: February 3

Compulsory. Closed book. Worth 35% of course grade.
Read question carefully so you answer the question.
Keep answers as brief as possible.

Question scores (total 35 points)

Question	1a	1b	1c	1d	1e	2a	2b	2c	2d	2e	3a	3b	3c	3d	3e	3f	4
Points	1	3	2	2	2	3	3	1	2	1	2	2	1	1	2	2	5

1. Consider the estimator $\widehat{\boldsymbol{\beta}}$ that minimizes

$$Q_N(\boldsymbol{\beta}) = \frac{1}{N}\sum\nolimits_{i=1}^{N}(y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}))^2,$$

where $\boldsymbol{\beta}$ is a $k \times 1$ parameter vector and $\mathbf{x}_i$ is a $k \times 1$ stochastic regressor vector and it is assumed that $(y_i, \mathbf{x}_i)$ are i.i.d. over i.
In the true model

$$\begin{aligned} \mathrm{E}[y_i|\mathbf{x}_i] &= \exp(\mathbf{x}_i'\boldsymbol{\beta}_0) \\ \mathrm{V}[y_i|\mathbf{x}_i] &= \sigma_{i0}^2, \text{ where } \sigma_{i0}^2 \text{ may depend on } \mathbf{x}_i \end{aligned}$$

If you need to make other assumptions, state them as you go along.
You can apply laws of large numbers and central limit theorems without formally verifying necessary conditions for their use (except 1(d) requires further detail).
You need not verify any second-order conditions.

(a) Show that $\widehat{\boldsymbol{\beta}}$ equivalently minimizes

$$\begin{aligned} Q_N(\boldsymbol{\beta}) &= \frac{1}{N}\sum_{i=1}^{N}\{[y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}_0)]^2 + 2[y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}_0)][\exp(\mathbf{x}_i'\boldsymbol{\beta}_0) - \exp(\mathbf{x}_i'\boldsymbol{\beta})] \\ &\quad +[\exp(\mathbf{x}_i'\boldsymbol{\beta}_0) - \exp(\mathbf{x}_i'\boldsymbol{\beta})]^2\}. \end{aligned}$$

(b) Using the expression in part (a) (even if you couldn't obtain it) obtain $Q_0(\boldsymbol{\beta}) = \operatorname{plim} Q_N(\boldsymbol{\beta})$.
Hint: $\mathrm{E}[[y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta}_0)]^2|\mathbf{x}_i] = \mathrm{V}[y_i|\mathbf{x}_i]$ since $\mathrm{E}[y_i|\mathbf{x}_i] = \exp(\mathbf{x}_i'\boldsymbol{\beta}_0)$.

(c) Using (b) prove that the local minimum of $Q_N(\boldsymbol{\beta})$ is consistent for $\boldsymbol{\beta}_0$. State any assumptions made.

(d) Now state what LLN you would use to verify part (b) and what additional information, if any, is needed to apply this law. A brief answer will do. There is no need for a formal proof.

(e) If the functional form for $\mathrm{E}[y_i|\mathbf{x}_i]$ in this example is misspecified will $\widehat{\boldsymbol{\beta}}$ still be consistent? Explain.

2. Continue with the estimator in question 1.

(a) Give the limit distribution of $\sqrt{N}\frac{\partial Q_N}{\partial \beta}\Big|_{\boldsymbol{\beta}_0}$, assuming that the usual asymptotic result applies.

(b) Obtain the probability limit of $\frac{\partial^2 Q_N}{\partial \beta \partial \beta'}\Big|_{\boldsymbol{\beta}_0}$, assuming that the usual asymptotic result applies.

(c) Combine the above to obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.

(d) Provide the formula for a consistent estimate of the variance covariance matrix of $\widehat{\boldsymbol{\beta}}$.

(e) Hence provide a test of $H_0 : \beta_j = 0$ against $H_a : \beta_j \neq 0$ at level 0.05.

3.(a) Provide a formal definition of convergence in probability.

(b) Provide a formal definition of convergence in distribution.

(c) If a sequence $b_N \xrightarrow{d} b$, will $b_N \xrightarrow{p} b$?

(d) Why are laws of large numbers used so often in econometrics?

(e) Provide the formula for the Newton-Raphson method for computation of an estimator $\widehat{\boldsymbol{\theta}}$ that maximizes the function $Q_N(\boldsymbol{\theta})$ for general $\boldsymbol{\theta}$.

(f) Derive the log-likelihood function for the logit model.

4. Estimation of a mystery model using a mystery method yields the output given below. It is known that $\mathrm{E}[y|x_2, x_3] = g(\beta_1 + \beta_2 x_2 + \beta_3 x_3)$ where the functional form $g(\cdot)$ is unknown, but it is known that $g(\cdot)$ is monotonic decreasing.

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. sum y x2 x3
    Variable |       Obs        Mean    Std. Dev.
           y |       167    .3473054    .6202229
          x2 |       167    .0443812    1.020856
          x3 |       167    .0263457    .5186052

Mystery regression                                Number of obs   =        167
                                                  chi2(2)         =       6.87
                                                  Prob > chi2     =     0.0322
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x2 |    .272362   .1331826     2.05   0.041     .0113289    .5333952
          x3 |   .4223453   .2653977     1.59   0.112    -.0978247    .9425153
       _cons |  -1.141784   .1426118    -8.01   0.000    -1.421298   -.8622702
------------------------------------------------------------------------------
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Provide, with explanation, as detailed an interpretation as you can of the output.
[Note: This question is worth five points].