240D Cameron Winter 2012
Department of Economics, U.C.-Davis

Midterm Exam: February 9

Compulsory. Closed book. Worth 35% of course grade.
Read question carefully so you answer the question.
Keep answers as brief as possible.

Question scores (total 35 points)

Question	1a	1b	1c	1d	1e	2a	2b	2c	2d	2e	3a	3b	3c	3d	4a	4b	4c
Points	2	3	2	2	2	2	3	2	2	2	2	2	2	2	2	2	1

1. If y takes only non-negative integer values and has geometric density with parameter λ then the density $f(y|\lambda)$ is

$$f(y) = \lambda^y (1+\lambda)^{-(y+1)}, \quad y = 0, 1, ..., \quad \lambda > 0.$$

Furthermore $\mathrm{E}[y] = \lambda$ and $\mathsf{V}[y] = \lambda(1+\lambda)$.

Here we introduce regressors and suppose that in the true model the parameter λ depends on regressors according to

$$\lambda_i = \exp(\mathbf{x}_i'\boldsymbol{\beta}),$$

where $\boldsymbol{\beta}$ is an unknown $K \times 1$ parameter vector and $\mathbf{x}_i$ is a nonstochastic $K \times 1$ vector.

In the dgp $\boldsymbol{\beta} = \boldsymbol{\beta}_0$, and data are independent over i.

If you need to make other assumptions, state them as you go along.
You can apply laws of large numbers and central limit theorems without formally verifying necessary conditions for their use (except 1(d) requires further detail).
You need not verify any second-order conditions.

(a) Show that the MLE for $\boldsymbol{\beta}$ maximizes (upon scaling by N^{-1})

$$Q_n(\beta) = N^{-1} \sum_{i=1}^{N} y_i \mathbf{x}_i'\boldsymbol{\beta} - (y_i + 1)\ln(1 + \exp(\mathbf{x}_i'\boldsymbol{\beta})).$$

(b) Obtain $Q_0(\boldsymbol{\beta}) = \operatorname{plim} Q_N(\boldsymbol{\beta})$.

(c) Using (b) prove that the local max to $Q_N(\boldsymbol{\beta})$ is consistent for $\boldsymbol{\beta}_0$. State any assumptions made.

(d) Now state what LLN you would use to verify part (b) and what additional information, if any, is needed to apply this law. A brief answer will do. There is no need for a formal proof.

(e) If the density $f(y)$ is misspecified will the MLE in this example be consistent? Explain.

2.(a) Show that the derivative with respect to $\boldsymbol{\beta}$ can be written as

$$\frac{1}{N}\sum_i \left(\frac{y_i - \exp(\mathbf{x}_i'\boldsymbol{\beta})}{1+\exp(\mathbf{x}_i'\boldsymbol{\beta})}\right)\mathbf{x}_i.$$

(b) Give the limit distribution of $\sqrt{N}\frac{\partial Q_N}{\partial \boldsymbol{\beta}}\Big|_{\boldsymbol{\beta}_0}$.

(c) Obtain the probability limit of $\frac{\partial^2 Q_N}{\partial \boldsymbol{\beta}\partial \boldsymbol{\beta}'}\Big|_{\boldsymbol{\beta}_0}$.

(d) Combine the above to obtain the limit distribution of $\sqrt{N}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0)$.

(e) Hence provide a test of $H_0 : \beta_j = 0$ against $H_a : \beta_j \neq 0$ at level 0.05.

3.(a) Provide a formal definition of convergence in probability.

(b) Give an example of a central limit theorem in the scalar case, stating clearly the assumptions of the central limit theorem and the conclusion.

(c) Give the general formula for a gradient method to obtain $\widehat{\boldsymbol{\theta}}$ that maximizes $Q_N(\boldsymbol{\theta})$.

(d) Derive the formula for the Newton-Raphson method.

4. Consider the following Stata code:

```
set obs 1000
set seed 10101
generate x = rnormal(0,1)
generate y = exp(x) + rnormal(0,1)
poisson y x, vce(robust)
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(a) What is the data generating process for this model?

(b) Do you see any problems (inconsistency, wrong standard errors, inefficiency, ...) in the Poisson estimates that will be obtained?

(c) Suppose estimation yields a slope coefficient of 0.9. How do we interpret the value 0.9?