Assignment 4: Nonlinear (b11)
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Data are at http://cameron.econ.ucdavis.edu/bgpe2011

1. Use data in file mus10data.dta and 2002 data (keep if year02==1)
We will do analysis similar to that in the slides, but with the two regressors chronic and age.
Note: Variable age is measured in tens of years!

(a) Perform Poisson regression of docvis on chronic and age, with heteroskedastic-robust standard errors computed. Are the regressors individually and jointly statistically significant at 5%?

(b) Use command test to test the hypothesis that one hundred years of aging has the same impact on doctor visits as having one (or more) chronic conditions.

(c) Use Stata add-on command margins dydx(•) to compute the average effect on the number of doctor visits of aging ten years.

(d) Use command margins dydx(•), atmean to compute the effect of aging ten years on the number of doctor visits for an individual with average values of the regressors.

(e) Use command margins dydx(•), at(•) to compute the effect of aging ten years on the number of doctor visits for a 50-year old with a chronic condition.

(f) Compare your answer in parts (c) and (d) to the average marginal effect obtained from OLS estimation of docvis on chronic and age.

(g) For the Poisson model, $E[y|x] = \exp(x^\prime \beta)$ ⇒ $\partial E[y|x]/\partial x = \exp(x^\prime \beta) \Rightarrow [\partial E[y|x]/\partial x]/E[y|x] = \beta$. Using this result, what is the estimated proportional impact of aging ten years on doctor visits?

2. If $y$ takes only non-negative integer values and has geometric density with parameter $\lambda$ then the density $f(y|\lambda)$ is

$$f(y) = \lambda^y (1 + \lambda)^{-(y+1)}, \quad y = 0, 1, \ldots, \lambda > 0.$$ 

Furthermore $E[y] = \lambda$ and $V[y] = \lambda(1 + \lambda)$.

Here we introduce regressors and suppose that in the true model the parameter $\lambda$ depends on regressors according to

$$\lambda_i = E[y_i|x_i] = \exp(x_i^\prime \beta),$$

where $\beta$ is an unknown $K \times 1$ parameter vector and $x_i$ is a nonstochastic $K \times 1$ vector.

The data are assumed to be independent over $i$.

(a) Show that the log-likelihood function for this example is

$$Q(\beta) = \sum_{i=1}^N \{y_i x_i^\prime \beta - (y_i + 1) \ln(1 + \exp(x_i^\prime \beta))\}.$$ 

(b) Show that the first-order conditions for the geometric MLE can be simplify to

$$\frac{\partial Q(\beta)}{\partial \beta} = \sum_{i=1}^N \left( \frac{y_i - \exp(x_i^\prime \beta)}{1 + \exp(x_i^\prime \beta)} \right) x_i = 0.$$ 

(c) Given this result, do you think that correct specification of the conditional mean will be sufficient to ensure that the MLE is consistent? Give a brief explanation.

(d) Show that

$$E\left[ \frac{\partial^2 Q(\beta)}{\partial \beta^2} | x_1, \ldots, x_N \right] = \sum_{i=1}^N \frac{-\exp(x_i^\prime \beta)}{1 + \exp(x_i^\prime \beta)} x_i x_i^\prime.$$ 

(e) Give the asymptotic distribution for the geometric MLE.
3. When $\gamma = h(\beta)$ where $\gamma$ and $\beta$ are scalars, the delta method yields $V^2 = \left(\frac{\partial \gamma}{\partial \beta}\right)^2 V^2$.

(a) Hence give the formula for a 95% confidence interval for $e^\beta$ given knowledge of $\hat{\beta}$ and $se(\hat{\beta})$.

(b) Compute this for $e^{\beta_{\text{age}}}$ following the Poisson regression in question 1.

(c) Compare your answer to that from post-estimation command `nlcom exp(_b[age])`.

4. Stata command `ml` can be used to compute the MLE for a user-provided log-likelihood function. The following code does this for the Poisson MLE.

* 1. Poisson ML program `lfpois` defines the log-likelihood function (`lnf`)

```stata
program define lfpois, version 10.0
  args lnf theta1 // theta1=x'b, lnf=lnf(y)
  tempvar lnyfact mu
  local y "$ML_y1" // Define y and mu so program is more readable
  generate double 'mu' = exp('theta1')
  quietly replace 'lnf' = -'mu' + 'y'*'theta1' - lnfactorial('y')
end
```

* 2. Command `ml model` defines y and x, and here asks for robust se's

```stata
ml model lf lfpois (docvis = chronic age), vce(robust)
```

* 3. Command `ml maximize` computes the estimator

```stata
ml maximize
```

**Important:** Note that ‘’ is left quote ‘ and right quote ‘.
You need not understand the syntax in `lfpois` - just type out as is.

[Aside: `args` defines the program arguments; `tempvar` defines a temporary variable just used in this program; `local` defines a local macro; and the `args`, `tempvar` and `local` are from thereon referred to in single quotes as they are local to this program. The name of the dependent variable (in this example `docvis`) is stored in the global macro `ML_y1`. The global macro is then referred to with prefix $. In the program we define the local macro `y` to be text for the name of the global macro (hence the double quotes).]

(a) Run this program.

(b) Compare your results to those obtained using command `Poisson`.

5.(a) Use Stata command `ml` to estimate the MLE for the geometric regression of `docvis` on `chronic` and `age`. This is exactly the same as the program `lfpois` in question 4, except that you will need to replace the line

```stata
quietly replace 'lnf' = -'mu' + 'y'*'theta1' - lnfactorial('y')
```

with the corresponding expression for the log-density of the geometric (from question 2).

(b) Compare your results to those obtained using command `Poisson`.

(c) Which model do you prefer on the basis of fitted likelihood value: Poisson or geometric?

(d) Command `ml` uses the Newton-Raphson iterative method. Use your answers in question 2 to give the algebraic expressions for the Newton-Raphson method for the geometric. For simplicity use the expected Hessian $E[H_s]$ rather than the Hessian $H_s$. 

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