

# Panel Data Methods

## 2: Short Panels - Extensions

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*Based on A. Colin Cameron and Pravin K. Trivedi (2009, 2010),  
Microeconometrics using Stata (MUS), Stata Press.  
and A. Colin Cameron and Pravin K. Trivedi (2005),  
Microeconometrics: Methods and Applications (MMA), C.U.P.*

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# 1. Introduction

- Go beyond basic fixed effects and random effects.
- Panel GMM estimation
  - ▶ IV estimator
  - ▶ Hausman Taylor estimator
  - ▶ Arellano-Bond estimator for dynamic models.
- Panel Random Slopes models
  - ▶ Mixed Linear Models
- Panel estimation commands can be used for clustered data
  - ▶ e.g. Village level data
- Presentation follows Cameron and Trivedi (2010) chapter 9.

# Outline

- 1 Introduction
- 2 Panel GMM Theory
- 3 Linear Panel IV Estimation
- 4 Hausman-Taylor Estimator
- 5 Dynamic Models - Arellano Bond
- 6 Dynamic Models - Panel VAR
- 7 Mixed Linear Models
- 8 Clustered Data
- 9 Summary

## 2. Panel GMM Estimator

- Short panel: Independent over  $i$ ,  $T$  fixed,  $N \rightarrow \infty$ .
- Linear panel model

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + u_{it}.$$

- Stack all  $T$  observations for the  $i^{\text{th}}$  individual,

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{u}_i,$$

where  $\mathbf{y}_i$  and  $\mathbf{u}_i$  are  $T \times 1$  vectors and  $\mathbf{X}_i$  is  $T \times K$  with  $t^{\text{th}}$  row  $\mathbf{x}'_{it}$ , so

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{iT} \end{bmatrix}; \quad \mathbf{X}_i = \begin{bmatrix} \mathbf{x}'_{i1} \\ \vdots \\ \mathbf{x}'_{iT} \end{bmatrix}; \quad \mathbf{u}_i = \begin{bmatrix} u_{i1} \\ \vdots \\ u_{iT} \end{bmatrix}.$$

- The model  $\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{u}_i$  is a linear system of equations, so systems IV estimation.

## Panel GMM Estimator (continued)

- Assume existence of a  $T \times r$  matrix of instruments  $\mathbf{Z}_i$  that satisfy

$$E[\mathbf{Z}'_i \mathbf{u}_i] = \mathbf{0}.$$

- $r \geq K$  is the number of instruments.
- GMM estimator minimizes the associated quadratic form

$$Q_N(\beta) = \left[ \sum_{i=1}^N \mathbf{z}'_i \mathbf{u}_i \right]' \mathbf{W}_N \left[ \sum_{i=1}^N \mathbf{z}'_i \mathbf{u}_i \right],$$

- where  $\mathbf{W}_N$  denotes an  $r \times r$  weighting matrix.
- Given  $\mathbf{u}_i = \mathbf{y}_i - \mathbf{X}_i \beta$ , some algebra yields the panel GMM estimator

$$\begin{aligned} \hat{\beta}_{\text{PGMM}} &= \left[ \left( \sum_{i=1}^N \mathbf{x}'_i \mathbf{z}_i \right) \mathbf{W}_N \left( \sum_{i=1}^N \mathbf{z}'_i \mathbf{x}_i \right) \right]^{-1} \\ &\quad \times \left( \sum_{i=1}^N \mathbf{x}'_i \mathbf{z}_i \right) \mathbf{W}_N \left( \sum_{i=1}^N \mathbf{z}'_i \mathbf{y}_i \right). \end{aligned}$$

- The essential condition for consistency of this estimator is  $E[\mathbf{Z}'_i \mathbf{u}_i] = \mathbf{0}$ .

## Instruments

- Usually need instrument  $\mathbf{z}$  to be some variable other than  $\mathbf{x}$ .
- Here, however, if only current period  $\mathbf{x}_{it}$  belong in the model for  $y_{it}$  then  $\mathbf{x}'_{it}$ s from other periods may become available as instruments.
- Contemporaneous exogeneity assumption: it is only current  $\mathbf{z}$  and  $\varepsilon$  that are uncorrelated

$$E[\mathbf{z}_{it}u_{it}] = \mathbf{0}, \quad t = 1, \dots, T,$$

- ▶ current  $\mathbf{z}_{it}$  only are available as instruments
- Weak exogeneity assumption (predetermined instruments) assumption: lagged values of  $\mathbf{z}$  are uncorrelated with the current period error

$$E[\mathbf{z}_{is}u_{it}] = \mathbf{0}, \quad s \leq t, \quad t = 1, \dots, T.$$

- ▶  $\mathbf{z}_{i1}, \dots, \mathbf{z}_{it}$  can be instruments for  $u_{it}$ , though future values of  $\mathbf{z}_{is}$  cannot be so used.

## Instruments (continued)

- Strict exogeneity assumption future values of instruments are also uncorrelated with the current period error

$$E[\mathbf{z}_{is} u_{it}] = \mathbf{0}, \quad s, t = 1, \dots, T.$$

- ▶ Current, past and future values of  $\mathbf{z}_{is}$  are valid instruments for  $u_{it}$
- Strict exogeneity is assumption made to date
  - ▶  $E[u_{it} | \mathbf{x}_{j1}, \dots, \mathbf{x}_{jT}] = 0$  implies  $E[u_{it} | \mathbf{x}_{is}] = 0$ ,  $1 \leq s \leq T$ , and hence  $E[\mathbf{x}_{is} u_{it}] = \mathbf{0}$ .
  - ▶ appropriate for static models
  - ▶ for dynamic models at most weak exogeneity of instruments can be assumed.
- So many instruments may be available (over-identified)
  - ▶ though time-invariant regressors can only be used as an instrument once.

# GMM Estimator Distribution

- Rewrite

$$\hat{\beta}_{\text{PGMM}} = [\mathbf{X}'\mathbf{Z}\mathbf{W}_N\mathbf{Z}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{Z}\mathbf{W}_N\mathbf{Z}'\mathbf{y},$$

- ▶ where  $\mathbf{X}' = [\mathbf{x}'_1 \cdots \mathbf{x}'_N]$ ,  $\mathbf{Z}' = [\mathbf{z}'_1 \cdots \mathbf{z}'_N]$ ,  $\mathbf{y}' = [\mathbf{y}'_1 \cdots \mathbf{y}'_N]$ .

- Then  $\hat{\beta}_{\text{PGMM}}$  is asymptotically normal with

$$\hat{V}[\hat{\beta}_{\text{PGMM}}] = [\mathbf{X}'\mathbf{Z}\mathbf{W}_N\mathbf{Z}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{Z}\mathbf{W}_N(N\hat{\mathbf{S}})\mathbf{W}'_N\mathbf{Z}'\mathbf{X}[\mathbf{X}'\mathbf{Z}\mathbf{W}_N\mathbf{Z}'\mathbf{X}]^{-1},$$

- Given independence over  $i$ ,  $\hat{\mathbf{S}}$  is consistent estimate of the  $r \times r$  matrix

$$\mathbf{S} = \text{plim} \frac{1}{N} \sum_{i=1}^N \mathbf{z}'_i \mathbf{u}_i \mathbf{u}'_i \mathbf{z}_i.$$

- A White-type panel robust estimate of  $\mathbf{S}$  is

$$\hat{\mathbf{S}} = \frac{1}{N} \sum_{i=1}^N \mathbf{z}'_i \hat{\mathbf{u}}_i \hat{\mathbf{u}}'_i \mathbf{z}_i,$$

- ▶ where the  $T \times 1$  estimated residual  $\hat{\mathbf{u}}_i = \mathbf{y}_i - \mathbf{X}_i \hat{\beta}$
- ▶ controls for arbitrary heteroskedasticity and correlation
- ▶ can panel jackknife or panel bootstrap to get this.

# One-step and Two-step GMM

- One-step GMM or two-stage least squares estimator
  - ▶  $\mathbf{W}_N = [\sum_i \mathbf{Z}'_i \mathbf{Z}_i]^{-1} = [\mathbf{Z}'\mathbf{Z}]^{-1}$
  - ▶  $\hat{\beta}_{2SLS} = [\mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{y}$ .
  - ▶ Optimal PGMM estimator based on  $E[\mathbf{Z}'_i \mathbf{u}_i] = \mathbf{0}$  if  $\mathbf{u}_i | \mathbf{Z}_i$  is iid  $[\mathbf{0}, \sigma^2 \mathbf{I}_T]$ .
  
- Two-step GMM estimator
  - ▶  $\mathbf{W}_N = \hat{\mathbf{S}}^{-1}$  where  $\hat{\mathbf{S}}$  is consistent for  $\mathbf{S}$
  - ▶  $\hat{\beta}_{2SGMM} = [\mathbf{X}'\mathbf{Z}\hat{\mathbf{S}}^{-1}\mathbf{Z}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{Z}\hat{\mathbf{S}}^{-1}\mathbf{Z}'\mathbf{y}$
  - ▶ Optimal PGMM estimator based on  $E[\mathbf{Z}'_i \mathbf{u}_i] = \mathbf{0}$  given independence over  $i$
  - ▶  $\hat{\mathbf{S}}$  Comes from residuals after one-step 2SLS estimation.

## Overidentifying Restrictions Test

- $r$  instruments  $> K$  parameters  
 $\Rightarrow (r - K)$  overidentifying (or extra) restrictions
  - ▶ test whether  $\sum_{i=1}^N \mathbf{Z}'_i \hat{\mathbf{u}}_i$  is close to 0.
- After two-step GMM (not one-step) test of overidentifying restrictions (Sargan or Hansen test) is

$$\tau_N = \left[ \sum_{i=1}^N \hat{\mathbf{u}}'_i \mathbf{Z}_i \right] (N\hat{\mathbf{S}})^{-1} \left[ \sum_{i=1}^N \mathbf{Z}'_i \hat{\mathbf{u}}_i \right]$$

- ▶ where  $\hat{\mathbf{u}}_i = \mathbf{y}_i - \mathbf{Z}'_i \hat{\beta}_{2SGMM}$
- $\tau_N \sim \chi^2(r - K)$  under  $H_0$  : over-identifying restrictions are valid.
- If  $\tau_N$  is large overidentifying moment conditions are rejected
  - ▶ conclude some of the instruments in  $\mathbf{Z}_i$  are correlated with the error.
- If  $\tau_N$  is small overidentifying moment conditions are not rejected
  - ▶ conclude that, conditional on  $K$  linear combinations of the instruments being valid, cannot reject that the remaining  $(r - K)$  are not also valid.

### 3. Panel IV Estimation

- Consider model with possibly transformed variables:

$$y_{it}^* = \alpha + \mathbf{x}_{it}^* \boldsymbol{\beta} + u_{it}^*,$$

- Transformations are

OLS	$y_{it}^* = y_{it}$
Between	$y_{it}^* = \bar{y}_i$
Fixed effects	$y_{it}^* = (y_{it} - \bar{y}_i)$
Random effects	$y_{it}^* = (y_{it} - \theta_i \bar{y}_i)$

- OLS is **inconsistent** if  $E[u_{it}^* | \mathbf{x}_{it}^*] \neq 0$ .
- IV estimation** with **instruments**  $\mathbf{z}_{it}^*$  (also transformed) that satisfy  $E[u_{it}^* | \mathbf{z}_{it}^*] = 0$ .
- Stata command `xtivreg` does this
  - natural extension of `ivregress` to panels
  - though only does 2SLS and not GMM.

## IV Fixed Effects Example

- Regress  $\ln \text{wage}$  on  $\text{experience}$ ,  $\text{experience}^2$ , and  $\text{weeks worked}$ .
- Marital status instrument for  $\text{weeks worked}$ .
- Output on next slide:
  - ▶ Standard errors are more than 10 times larger than for regular FE
  - ▶ not unusual for IV
  - ▶ coefficients change sign though statistically insignificant.
- Standard errors assume iid errors
  - ▶ no `vce(robust)` option
  - ▶ instead use `vce(bootstrap)` or `vce(jackknife)`.

```
. * Panel IV example: FE with wks instrumented by external instrument ms
. use mus08psidextract.dta, clear
(PSID wage data 1976-82 from Baltagi and Khanti-Akom (1990))
```

```
. xtivreg lwage exp exp2 (wks = ms), fe
```

```
Fixed-effects (within) IV regression      Number of obs      =      4165
Group variable: id                       Number of groups   =      595

R-sq:  within = .
        between = 0.0172
        overall = 0.0284

Obs per group:  min =      7
                  avg =     7.0
                  max =      7

corr(u_i, Xb) = -0.8499

Wald chi2(3) = 700142.43
Prob > chi2 = 0.0000
```

	lwage	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
	wks	-.1149742	.2316926	-0.50	0.620	-.5690832	.3391349
	exp	.1408101	.0547014	2.57	0.010	.0335974	.2480228
	exp2	-.0011207	.0014052	-0.80	0.425	-.0038748	.0016334
	_cons	9.83932	10.48955	0.94	0.348	-10.71983	30.39847
	sigma_u	1.0980369					
	sigma_e	.51515503					
	rho	.81959748	(fraction of variance due to u_i)				

```
F test that all u_i=0:      F(594,3567) =      4.62      Prob > F = 0.0000
```

```
Instrumented:  wks
Instruments:  exp exp2 ms
```

## 4. Hausman-Taylor IV Estimator

- We want to estimate the coefficient of a time-invariant regressor.
- Problem: the fixed effect estimator is not identified for the-invariant regressor
- Solution: assume that some time-varying regressors are not correlated with either  $\alpha_j$  or  $\varepsilon_{it}$ 
  - ▶ then their values in periods other than the current period can be used as instruments
  - ▶ called the Hausman-Taylor estimator
- Stata command `xthtaylor` implements this
  - ▶ for panel-robust se's use `vce(bootstrap)` or `vce(jackknife)`.

# Hausman-Taylor IV Estimator (continued)

- Hausman and Taylor (1981) model

$$y_{it} = \mathbf{x}'_{1it}\boldsymbol{\beta}_1 + \mathbf{x}'_{2it}\boldsymbol{\beta}_2 + \mathbf{w}'_{1i}\boldsymbol{\gamma}_1 + \mathbf{w}'_{2i}\boldsymbol{\gamma}_2 + \alpha_i + \varepsilon_{it},$$

- ▶  $\mathbf{w}$  is introduced to denote time-invariant regressors
- Assume
  - ▶  $\mathbf{x}_{1it}$  and  $\mathbf{w}_{1i}$  are uncorrelated with  $\alpha_i$
  - ▶  $\mathbf{x}_{2it}$  and  $\mathbf{w}_{2i}$  are correlated with  $\alpha_i$
  - ▶ all regressors are assumed to be uncorrelated with  $\varepsilon_{it}$ .
- Then  $\mathbf{x}_{1is}$  for  $s \neq t$  available as instruments.
  - ▶ can test validity of overidentifying restrictions using OIR test.

## Hausman-Taylor IV Estimator (continued)

- Actually assume a RE error model and do more efficient IV
  - ▶ first transform model as in FGLS

$$\tilde{y}_{it} = \tilde{\mathbf{x}}'_{1it}\boldsymbol{\beta}_1 + \tilde{\mathbf{x}}'_{2it}\boldsymbol{\beta}_2 + \tilde{\mathbf{w}}'_{1i}\boldsymbol{\gamma}_1 + \tilde{\mathbf{w}}'_{2i}\boldsymbol{\gamma}_2 + v_{it}$$

- ▶ where  $\tilde{\mathbf{x}}_{1it} = \mathbf{x}_{1it} - \lambda \bar{\mathbf{x}}_{1i}$  and  $\lambda = 1 - \sigma_\varepsilon / \sqrt{\sigma_\varepsilon^2 + T\sigma_\alpha^2}$ .
- Then the instruments are  $(\mathbf{x}_{1it} - \bar{\mathbf{x}}_{1i})$ ,  $(\mathbf{x}_{2it} - \bar{\mathbf{x}}_{2i})$ ,  $\mathbf{w}_{1i}$  and  $\bar{\mathbf{x}}_{1i}$ .
- Amemiya and MaCurdy (1986) propose additional instruments
  - ▶ option amacurdy
- Breusch et al (1989) propose even more.

# Hausman-Taylor Example

- Full model: we used subset of regressors so far
  - ▶ want to instruments for time-invariant endogenous regressor `ed`
  - ▶ assume time-varying regressors `occ`, `south`, `smsa` and `ind` (industry) are exogenous

```
. * Hausman-Taylor example of Baltagi and Khanti-Akom (1990)
. use mus08psidextract.dta, clear
(PSID wage data 1976-82 from Baltagi and Khanti-Akom (1990))

. xthtaylor lwage occ south smsa ind exp exp2 wks ms union fem blk ed, ///
> endog(exp exp2 wks ms union ed)
```

Hausman-Taylor estimation	Number of obs	=	4165
Group variable: id	Number of groups	=	595
	Obs per group: min	=	7
	avg	=	7
	max	=	7
Random effects $u_i \sim i.i.d.$	wald chi2(12)	=	6891.87
	Prob > chi2	=	0.0000

- Coefficient of ed now 0.138 (was 0.112 with xtreg, re)
  - ▶ se of ed increases to 0.084 from 0.0212.

lwage	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
TVexogenous						
occ	-.0207047	.0137809	-1.50	0.133	-.0477149	.0063055
south	.0074398	.031955	0.23	0.816	-.0551908	.0700705
smsa	-.0418334	.0189581	-2.21	0.027	-.0789906	-.0046761
ind	.0136039	.0152374	0.89	0.372	-.0162608	.0434686
TVendogenous						
exp	.1131328	.002471	45.79	0.000	.1082898	.1179758
exp2	-.0004189	.0000546	-7.67	0.000	-.0005259	-.0003119
wks	.0008374	.0005997	1.40	0.163	-.0003381	.0020129
ms	-.0298508	.01898	-1.57	0.116	-.0670508	.0073493
union	.0327714	.0149084	2.20	0.028	.0035514	.0619914
TIexogenous						
fem	-.1309236	.126659	-1.03	0.301	-.3791707	.1173234
blk	-.2857479	.1557019	-1.84	0.066	-.5909179	.0194221
TIendogenous						
ed	.137944	.0212485	6.49	0.000	.0962977	.1795902
_cons	2.912726	.2836522	10.27	0.000	2.356778	3.468674
sigma_u	.94180304					
sigma_e	.15180273					
rho	.97467788	(fraction of variance due to u_i)				

Note: TV refers to time varying; TI refers to time invariant.

## 5. Linear Dynamic Panel Models

- Simple dynamic model regresses  $y_{it}$  in **polynomial in time**.
  - ▶ e.g. Growth curve of child height or IQ as grow older
  - ▶ use previous models with  $\mathbf{x}_{it}$  polynomial in time or age.
- Richer dynamic model regresses  $y_{it}$  on **lags** of  $y_{it}$ .

# Linear Dynamic Panel Models with Individual Effects

- **Leading example:** AR(1) model with individual specific effects

$$y_{it} = \alpha_i + \gamma y_{i,t-1} + \mathbf{x}'_{it} \boldsymbol{\beta} + \varepsilon_{it}.$$

- Four reasons for  $y_{it}$  being serially correlated over time:
  - ▶ **Unobserved heterogeneity:** via  $\alpha_i$  (permanent)
  - ▶ **True state dependence:** via  $y_{i,t-1}$  (dampens over time)
  - ▶ **Observed heterogeneity:** via  $\mathbf{x}_{it}$  which may be serially correlated
  - ▶ **Error correlation:** via  $\varepsilon_{it}$
- There is a literature on dynamic models with random effects.
- We focus on case where  $\alpha_i$  is a **fixed effect**
  - ▶ and  $T$  small so cannot consistently estimate  $\alpha_i$
  - ▶ no problem if  $T \rightarrow \infty$  and  $\varepsilon_{it}$  serially uncorrelated.

# Fixed Effects Estimator is Inconsistent

- Mean difference yields

$$(y_{it} - \bar{y}_i) = \gamma(y_{i,t-1} - \bar{y}_{i,-1}) + (\mathbf{x}_{it} - \bar{\mathbf{x}}_i)' \boldsymbol{\beta} + (\varepsilon_{it} - \bar{\varepsilon}_i).$$

- Problem: regressor  $(y_{i,t-1} - \bar{y}_{i,-1})$  correlated with  $(\varepsilon_{it} - \bar{\varepsilon}_i)$ 
  - ▶ since  $y_{it}$  (part of  $\bar{y}_{i,-1}$ ) is correlated with  $\varepsilon_{it}$
  - ▶ and  $y_{i,t-1}$  is correlated with  $\varepsilon_{it}$  (part of  $\bar{\varepsilon}_i$ ).
- This inconsistency is called Nickell bias
  - ▶ inconsistency is  $O(T^{-1})$ .

## Arellano-Bond Estimator

- **First-difference** to eliminate  $\alpha_i$  (rather than mean-difference)

$$(y_{it} - y_{i,t-1}) = \gamma(y_{i,t-1} - y_{i,t-2}) + (\mathbf{x}_{it} - \mathbf{x}'_{i,t-1})\boldsymbol{\beta} + (\varepsilon_{it} - \varepsilon_{i,t-1}).$$

- **OLS inconsistent** as  $(y_{i,t-1} - y_{i,t-2})$  correlated with  $(\varepsilon_{it} - \varepsilon_{i,t-1})$  (even under assumption  $\varepsilon_{it}$  is serially uncorrelated).
- But  $y_{i,t-2}$  is uncorrelated with  $(\varepsilon_{it} - \varepsilon_{i,t-1})$ , **if  $\varepsilon_{it}$  serially uncorrelated**  
so can use  $y_{i,t-2}$  as an **instrument** for  $(y_{i,t-1} - y_{i,t-2})$ .
- Arellano-Bond is a variation that uses **unbalanced set** of instruments with further lags as instruments.  
For  $t = 3$  can use  $y_{i1}$ , for  $t = 4$  can use  $y_{i1}$  and  $y_{i2}$ , and so on.
- Stata commands
  - ▶ `xtabond` for Arellano-Bond
  - ▶ `xtdpdsys` for Blundell-Bond (more efficient than `xtabond`)
  - ▶ `xtdpd` for more complicated models than `xtabond` and `xtdpdsys`.

## Example Stata

```
. * Optimal or two-step GMM for a dynamic panel model
. xtabond lwage occ south smsa ind, lags(2) maxldep(3) ///
  pre(wks,lag(1,2)) endogenous(ms,lag(0,2)) ///
. endogenous(union,lag(0,2)) twostep vce(robust) ///
. artests(3)
. * Test whether error is serially correlated
. estat abond
. * Test of overidentifying restrictions
. estat sargan
. * Arellano/Bover or Blundell/Bond for dynamic panel
. xtdpdsys lwage occ south smsa ind, lags(2) ///
  maxldep(3) pre(wks,lag(1,2)) endog(ms,lag(0,2)) ///
  endogenous(union,lag(0,2)) twostep vce(robust) ///
. artests(3)
```

## Data Example: Pure Time Series AR(2)

- Pure AR(2):  $y_{it} = \gamma_1 y_{i,t-1} + \gamma_2 y_{i,t-2} + \Delta \varepsilon_{it}$ ,  $t = 1, 2, \dots, 7$ .
- Sufficient data to obtain IV estimates in the differenced model

$$\Delta y_{it} = \gamma_1 \Delta y_{i,t-1} + \gamma_2 \Delta y_{i,t-2} + \Delta \varepsilon_{it}, \quad t = 4, 5, 6, 7.$$

- Instruments
  - ▶ At  $t = 4$ : 2 available instruments  $y_{i1}$  and  $y_{i2}$  as uncorrelated with  $\Delta \varepsilon_{i4}$
  - ▶ At  $t = 5$ : 3 instruments,  $y_{i1}$ ,  $y_{i2}$  and  $y_{i3} \perp \Delta \varepsilon_{i5}$
  - ▶ At  $t = 6$ : 4 instruments  $y_{i1}, \dots, y_{i4}$
  - ▶ At  $t = 7$ : 5 instruments  $y_{i1}, \dots, y_{i5}$ .
- In all  $2 + 3 + 4 + 5 = 14$  instruments for  $\Delta y_{i,t-1}$  and  $\Delta y_{i,t-2}$ 
  - ▶ intercept is an instrument for itself.
- Estimation may be by 2SLS or by more efficient optimal GMM
  - ▶ instrument set is unbalanced: much easier to use command `xtabond` than it is to manually set up the instruments and use `ivregress`.

## • One-step GMM

```
. xtabond lwage, lags(2) vce(robust)
```

```
Arellano-Bond dynamic panel-data estimation   Number of obs   =   2380
Group variable: id                           Number of groups =   595
Time variable: t

Obs per group:   min =   4
                  avg =   4
                  max =   4

Number of instruments =   15                  wald chi2(2)    =  1253.03
                                                Prob > chi2     =   0.0000
```

One-step results

(Std. Err. adjusted for clustering on id)

lwage	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
lwage						
L1.	.5707517	.0333941	17.09	0.000	.5053005	.6362029
L2.	.2675649	.0242641	11.03	0.000	.2200082	.3151216
_cons	1.203588	.164496	7.32	0.000	.8811814	1.525994

Instruments for differenced equation

GMM-type: L(2/.)lwage

Instruments for level equation

Standard: \_cons

## Data Example: Pure Time Series (continued)

- There are  $4 \times 595 = 2380$  observations as lost  $t = 1, 2, 3$ .
- Results are reported in terms of the original levels model
  - ▶ even though mechanically the FD model is fitted.
- There are 15 instruments
  - ▶ output `L(2/. )` means that  $y_{i,t-2}, y_{i,t-3}, \dots, y_{i,1}$  are the instruments used for period  $t$ .
- Wages depend greatly on past wages, with the lag weights summing to  $.57 + .27 = .84$ .
- Why is there a constant term?
  - ▶ The estimated model actually has no constant term.
  - ▶ Instead the estimated constant coefficient is recovered using an auxiliary moment condition (Blundell-Bond levels - considered later).
  - ▶ `xtabond` gives same  $\hat{\gamma}$  and  $\hat{\beta}$  with or without `noconstant` option
  - ▶ advantage is want a constant if predict from levels equation.

- Two-step GMM - little change or efficiency gain

```
. * Optimal or two-step GMM for a pure time-series AR(2) panel model
. xtabond lwage, lags(2) twostep vce(robust)
```

```
Arellano-Bond dynamic panel-data estimation   Number of obs       =       2380
Group variable: id                             Number of groups    =         595
Time variable: t

Obs per group:   min =         4
                  avg =         4
                  max =         4

Number of instruments =       15                wald chi2(2)        =       1974.40
                                                Prob > chi2         =         0.0000
```

Two-step results

(Std. Err. adjusted for clustering on id)

lwage	Coef.	WC-Robust Std. Err.	z	P> z	[95% Conf. Interval]	
lwage						
L1.	.6095931	.0330542	18.44	0.000	.544808	.6743782
L2.	.2708335	.0279226	9.70	0.000	.2161061	.3255608
_cons	.9182262	.1339978	6.85	0.000	.6555952	1.180857

Instruments for differenced equation

GMM-type: L(2/.)lwage

Instruments for level equation

Standard: \_cons



## Data Example: Regressors

- More generally

$$y_{it} = \alpha_i + \sum_{j=1}^J \gamma_j y_{i,t-j} + \sum_{k=0}^K \mathbf{x}_{i,t-k}' \boldsymbol{\beta}_k + \varepsilon_{it}$$

$$\Delta y_{it} = \sum_{j=1}^J \gamma_j \Delta y_{i,t-j} + \sum_{k=0}^K \Delta \mathbf{x}_{i,t-k}' \boldsymbol{\beta}_k + \Delta \varepsilon_{it}.$$

- Regressors

- ▶ Drop time-invariant regressors: they are eliminated after first-differencing
- ▶ Lagged dependent: lag 2+ as instruments
  - ★ `lwage` appears as first two lags, as before
- ▶ Strictly exogenous  $E[x_{it}\varepsilon_{is}] = 0$  for all  $s, t$ : instruments for self
  - ★ `occ`, `south`, `smsa`, and `ind`
- ▶ Predetermined  $E[x_{it}\varepsilon_{is}] \neq 0$  for  $s < t$ : lag 1+ as instruments
  - ★ `wks` appears current and one lag
- ▶ Endogenous  $E[x_{it}\varepsilon_{it}] \neq 0$ : lag 2+ as instruments
  - ★ `ms` and `union`

## Data Example: Regressors (continued)

- Stata coding:

- ▶ strictly exogenous variables appear as regular regressors
- ▶ option `maxldep(3)` means at most three lags of  $y$  are used as instruments
- ▶ option `pre(wks,lag(1,2))` means `wks` and `L1.wks` are predetermined and only two additional lags are to be used as instruments.
- ▶ option `endogenous(ms,lag(0,2))` means endogenous `ms` appears only as a contemporaneous regressor and that at most two additional lags are used as instruments
- ▶ option `artests(3)` is needed for later `estat abond`

```
. * optimal or two-step GMM for a dynamic panel model
. xtabond lwage occ south smsa ind, lags(2) maxldep(3)      ///
> pre(wks,lag(1,2)) endogenous(ms,lag(0,2))                ///
> endogenous(union,lag(0,2)) twostep vce(robust) artests(3)
```

```
Arellano-Bond dynamic panel-data estimation   Number of obs       =       2380
Group variable: id                           Number of groups    =        595
Time variable: t

                                obs per group:   min =         4
                                                avg =         4
                                                max =         4
```

- Regressors  $x_{it}$  less statistically significant compared to static model.

Number of instruments = 40                      wald chi2(10) = 1287.77  
 Prob > chi2 = 0.0000

Two-step results

(Std. Err. adjusted for clustering on id)

lwage	Coef.	WC-Robust Std. Err.	z	P> z	[95% Conf. Interval]	
lwage						
L1.	.611753	.0373491	16.38	0.000	.5385501	.6849559
L2.	.2409058	.0319939	7.53	0.000	.1781989	.3036127
wks						
--.	-.0159751	.0082523	-1.94	0.053	-.0321493	.000199
L1.	.0039944	.0027425	1.46	0.145	-.0013807	.0093695
ms	.1859324	.144458	1.29	0.198	-.0972	.4690649
union	-.1531329	.1677842	-0.91	0.361	-.4819839	.1757181
occ	-.0357509	.0347705	-1.03	0.304	-.1038999	.032398
south	-.0250368	.2150806	-0.12	0.907	-.446587	.3965134
smsa	-.0848223	.0525243	-1.61	0.106	-.187768	.0181235
ind	.0227008	.0424207	0.54	0.593	-.0604422	.1058437
_cons	1.639999	.4981019	3.29	0.001	.6637377	2.616261

Instruments for differenced equation

GMM-type: L(2/4).lwage L(1/2).L.wks L(2/3).ms L(2/3).union

Standard: D.occ D.south D.smsa D.ind

Instruments for level equation

Standard: \_cons

## Test Error Correlation

- Preceding requires  $\varepsilon_{it}$  serially uncorrelated
  - ▶ otherwise lagged  $y$ 's no longer valid instrument.
- Test  $\varepsilon_{it}$  serially uncorrelated  $\Rightarrow \Delta\varepsilon_{it}$  is only MA(1)
  - ▶ test based on  $\Delta\hat{\varepsilon}_{it}$  from fitted model

```
. * Test whether error is serially correlated
. estat abond
```

Arellano-Bond test for zero autocorrelation in first-differenced errors

Order	z	Prob > z
1	-4.5244	0.0000
2	-1.6041	0.1087
3	.35729	0.7209

H0: no autocorrelation

- Only reject no autocorrelation in  $\Delta\varepsilon_{it}$  at order 1 (at level .05)
  - ▶ conclude we are okay as  $\Delta\varepsilon_{it}$  is only MA(1)
- Note: if e.g. MA(2) then add more lagged  $y$ 's to get to MA(1)
  - ▶ or use command `dpdsys` - allows e.g.  $\varepsilon_{it}$  is MA(1).

## Test Overidentifying Restrictions

- There are 40 instruments and 11 parameters
  - ▶ so 29 over-identifying restrictions.
- Do OIR test: only after 2step GMM and without `vce(robust)`

```
. * Test of overidentifying restrictions (first estimate with no vce(robust))
. quietly xtabond lwage occ south smsa ind, lags(2) maxldep(3) ///
> pre(wks,lag(1,2)) endogenous(ms,lag(0,2))          ///
> endogenous(union,lag(0,2)) twostep artests(3)

. estat sargan
Sargan test of overidentifying restrictions
H0: overidentifying restrictions are valid

      chi2(29)      =   39.87571
      Prob > chi2   =    0.0860
```

- Fail to reject  $H_0$  at level .05
  - ▶ conclude that the overidentifying restrictions are not rejected.
- If  $r - K$  is large there is bias for  $p \rightarrow 1$ .

# Arellano-Bover / Blundell-Bond “Systems GMM”

- Arellano-Bond instruments the **differenced model** using lags levels as instruments
  - ▶ based on e.g.  $E[y_{is}\Delta(\alpha_i + \varepsilon_{it})] = 0$  for  $s \leq t - 2$
- Can also instrument **levels model** using lag changes as instruments
  - ▶ based on e.g.  $E[\Delta y_{i,t-1}(\alpha_i + \varepsilon_{it})] = 0$ 
    - ★ an additional assumption:  $\Delta y_{i,t-1} \perp \alpha_i$  - deviations in  $y$  independent of fixed effect - see Blundell and Bond.
  - ▶ and also lagged once level of predetermined and endogenous
  - ▶ improves efficiency - intuition: model in levels has more info.

```
. * Arellano/Bover or Blundell/Bond for a dynamic panel model
> xtdpdsys l wage occ south smsa ind, lags(2) maxldep(3) ///
> pre(wks, lag(1,2)) endogenous(ms, lag(0,2)) ///
> endogenous(union, lag(0,2)) twostep vce(robust) artests(3)
```

```
System dynamic panel-data estimation      Number of obs      =      2975
Group variable: id                       Number of groups   =      595
Time variable: t                          Obs per group:    min =      5
                                              avg =      5
                                              max =      5

Number of instruments =      60            wald chi2(10)      =      2270.88
                                              Prob >_chi2       =      0.0000
```

- Here standard errors are 10%-60% smaller
  - ▶ though most coefficients much smaller
    - ★ levels model and differences model give different answers - problem
  - ▶ and extra 20 instruments used.

## Two-step results

	Coef.	WC-Robust Std. Err.	z	P> z	[95% Conf. Interval]	
<hr/>						
lwage						
lwage						
L1.	.6017533	.0291502	20.64	0.000	.5446199	.6588866
L2.	.2880537	.0285319	10.10	0.000	.2321322	.3439752
wks						
--.	-.0014979	.0056143	-0.27	0.790	-.0125017	.009506
L1.	.0006786	.0015694	0.43	0.665	-.0023973	.0037545
ms	.0395337	.0558543	0.71	0.479	-.0699386	.1490061
union	-.0422409	.0719919	-0.59	0.557	-.1833423	.0988606
occ	-.0508803	.0331149	-1.54	0.124	-.1157843	.0140237
south	-.1062817	.083753	-1.27	0.204	-.2704346	.0578713
smsa	-.0483567	.0479016	-1.01	0.313	-.1422422	.0455288
ind	.0144749	.031448	0.46	0.645	-.0471621	.0761118
_cons	.9584113	.3632287	2.64	0.008	.2464961	1.670327

## Instruments for differenced equation

GMM-type: L(2/4).lwage L(1/2).L.wks L(2/3).ms L(2/3).union

Standard: D.occ D.south D.smsa D.ind

## Instruments for level equation

GMM-type: LD.lwage LD.wks LD.ms LD.union

Standard: \_cons

- Use Arellano-Bond when  $N$  large  $T$  small
  - ▶ need large  $N$  as asymptotics in  $N$
  - ▶ keep number of instruments low
  - ▶ need small  $T$  as otherwise too many instruments, plus with larger  $T$  can just directly estimate fixed effects.
- Include time dummies, as require  $\varepsilon_{it}$  independent over  $i$  (not just  $t$ ).
- User-written add-on `xtabond2`
  - ▶ pre-dated Stata `xtabond`; still has some things that `xtabond` does not
  - ▶ forward orthogonal deviations (Helmert transformation) also eliminates  $\alpha_i$ 

$$y_{i,t=1}^+ = \sqrt{T/(T+1)}(y_{it} - \frac{1}{T} \sum_{s=t+1}^T y_{is})$$
    - ★ leads to fewer observations lost if unbalanced panel.
  - ▶ has a way to “collapse” instruments so less likely to have too many
  - ▶ David Roodman (2006): “How to Do `xtabond2`: An Introduction to “Difference” and “System” GMM in Stata” explains much about Arellano-Bond.

## 6. Short Panel Vector Autoregression

- Extend from Arellano-Bond for  $y_{it}$  to VAR for  $\mathbf{y}_{it}$  (several  $y$ 's)
  - ▶ Include individual specific effects  $\alpha$

$$\begin{aligned}\mathbf{y}_{it} &= \alpha_i + \mathbf{A}_1 \mathbf{y}_{i,t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{i,t-p} + \mathbf{u}_t \\ \mathbf{u}_t &\sim (\mathbf{0}, \Sigma),\end{aligned}$$

- Estimate by a systems version of Arellano and Bond (1992).
- This was actually considered in an earlier paper by Holtz-Eakin, D., W. Newey, and H.S. Rosen (1988), "Estimating Vector Autoregressions with Panel Data," *Econometrica*, 56, 1371-1395.
- Application: Love, I. and Ziccino, L. (2006), "Financial Development and Dynamic Investment Behaviour: Evidence from Panel VAR," *Quarterly Review of Economics and Finance*, 46, 190-210.

# Short Panel VAR Example

- Love has Stata user-written command: `pvar` (with `sgmm` and `helm`)
  - ▶ estimates model and graphs impulse-response functions
- As an example apply to Arellano-Bond data
  - ▶ 140 firms and 7 to 9 years of data
  - ▶ `n` is log employment
  - ▶ `k` is log capital
- First Helmert transform data
  - ▶ `use abdata.dta`
  - ▶ `helm n`
  - ▶ `helm k`
  - ▶ `pvar n k, lag(2) gmm impulse monte 50 decomp list_imp`

- Very high serial dependence

```
. pvar n k, lag(2) gmm impulse monte 50 decomp list_imp
GMM started : 15:33:18
accumulating matrices equation 1,2,calculating b2s1s
calculating big zuuz matrix
finished accumulating zuuz
_____ Results of the Estimation by system GMM_____
number of observations used : 611
-----
```

EQ1: dep.var : h\_n

	b_GMM	se_GMM	t_GMM
L.h_n	.96522745	.11722781	8.2337753
L.h_k	.17832941	.15812582	1.1277691
L2.h_n	-.12987744	.10193137	-1.2741655
L2.h_k	-.20248717	.0534318	-3.7896382

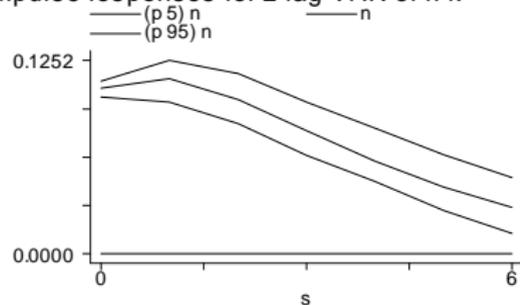
```
-----
```

EQ2: dep.var : h\_k

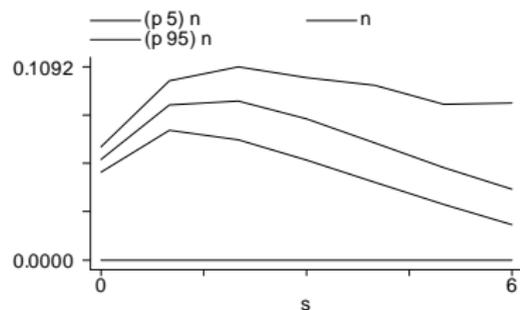
	b_GMM	se_GMM	t_GMM
L.h_n	.28801842	.13862581	2.077668
L.h_k	.99588169	.18395503	5.4137237
L2.h_n	-.16588097	.06990161	-2.3730636
L2.h_k	-.22096912	.05855934	-3.7734224

```
-----
just identified - Hansen statistic is not calculated
```

## Impulse-responses for 2 lag VAR of n k

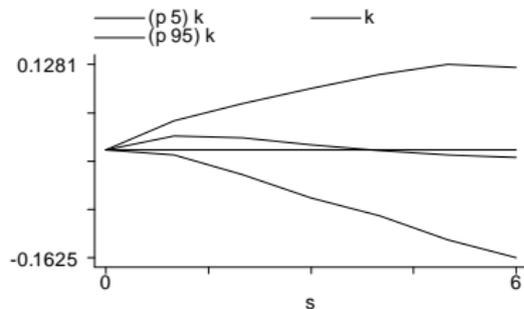


response of n to n shock

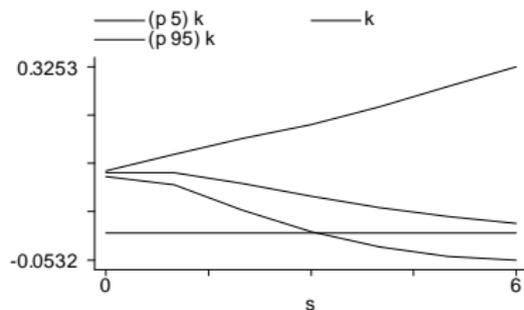


response of k to n shock

Errors are 5% on each side generated by Monte-Carlo with 50 reps



response of n to k shock



response of k to k shock

## 7. Mixed Linear Models: xtmixed and mixed

- Generalize random effects model to **random slopes**.
- Most applied statistics use mixed linear models
  - ▶ this is covered in detail below
  - ▶ xtmixed - Stata up to Stata 12
  - ▶ mixed - Stata 13 gives cluster-robust standard errors
- Econometricians also use **random coefficients model**

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta}_i + \varepsilon_{it},$$

- ▶  $(\alpha_i, \boldsymbol{\beta}_i)$  are iid with mean  $(\alpha, \boldsymbol{\beta})$  and variance matrix  $\Sigma$
- ▶  $\varepsilon_{it}$  is independent  $(0, \sigma_i^2)$  so heteroskedastic over  $i$  and can do FGLS
- ▶ command is xtrc. This is covered later.

## Mixed or Multi-level or Hierarchical Model: xtmixed

- Not used in microeconometrics but used in many other disciplines.
- Stack all observations for individual  $i$  and specify

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{u}_i + \varepsilon_i$$

where  $\mathbf{u}_i$  is iid  $(\mathbf{0}, \mathbf{G})$  and  $\mathbf{Z}_i$  is called a design matrix.

- Random effects:  $\mathbf{Z}_i = \mathbf{e}$  (a vector of ones) and  $\mathbf{u}_i = \alpha_i$
- Random coefficients:  $\mathbf{Z}_i = \mathbf{X}_i$ 
  - ▶ Reason:  $\boldsymbol{\beta}_i \sim (\boldsymbol{\beta}, \Sigma)$  so  $\boldsymbol{\beta}_i \sim \boldsymbol{\beta} + \mathbf{u}_i$  where  $\mathbf{u}_i \sim (\mathbf{0}, \Sigma)$
  - ▶ So  $\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta}_i + \varepsilon_i = \mathbf{X}_i(\boldsymbol{\beta} + \mathbf{u}_i) + \varepsilon_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{X}_i\mathbf{u}_i + \varepsilon_i$ .
  - ▶ Note:  $\mathbf{y}_i|\mathbf{X}_i$  has mean  $\mathbf{X}_i\boldsymbol{\beta}$  and variance  $\mathbf{X}_i'\Sigma\mathbf{X}_i + \sigma_\varepsilon^2\mathbf{I}$ .
- Example where just exp and wks have random slopes:

```
. xtmixed lwage exp exp2 wks ed || id: exp wks, ///
. covar(unstructured) mle
```

# Random Intercept

- The only random coefficient is that of the intercept
  - ▶ so RE model

```
. * Random intercept model estimated using xtmixed
. use mus08psidextract.dta, clear
(PSID wage data 1976-82 from Baltagi and Khanti-Akom (1990))

. xtmixed lwage exp exp2 wks ed || id:, mle
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0:  log likelihood = 293.69563
Iteration 1:  log likelihood = 293.69563
```

Computing standard errors:

```
Mixed-effects ML regression
Group variable: id
```

```
Number of obs      =      4165
Number of groups   =       595

Obs per group: min =         7
                avg =        7.0
                max =         7
```



## Random Slopes

- Allow intercept and slopes of `exp` and `wks` to vary across individuals.

```
. * Random-slopes model estimated using xtmixed
. xtmixed lwage exp exp2 wks ed || id: exp wks, covar(unstructured) mle
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0:  log likelihood = 397.61127 (not concave)
Iteration 1:  log likelihood = 427.28139 (not concave)
Iteration 2:  log likelihood = 481.60739 (not concave)
Iteration 3:  log likelihood = 493.48169
Iteration 4:  log likelihood = 495.95651
Iteration 5:  log likelihood = 508.23187
Iteration 6:  log likelihood = 509.00044
Iteration 7:  log likelihood = 509.00191
Iteration 8:  log likelihood = 509.00191
```

Computing standard errors:

```
Mixed-effects ML regression
Group variable: id
```

```
Number of obs      =      4165
Number of groups   =       595

obs per group: min =         7
                  avg =        7.0
                  max =         7
```

Log likelihood = 509.00191                      Prob > chi2                      =                      0.0000

l wage	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
exp	.0527159	.0032966	15.99	0.000	.0462546	.0591772
exp2	.0009476	.0000713	13.28	0.000	.0008078	.0010874
wks	.0006887	.0008267	0.83	0.405	-.0009316	.0023091
ed	.0868604	.0098652	8.80	0.000	.067525	.1061958
_cons	4.317674	.1420957	30.39	0.000	4.039171	4.596176

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
id: Unstructured				
sd(exp)	.043679	.0022801	.0394311	.0483846
sd(wks)	.0081818	.0008403	.00669	.0100061
sd(_cons)	.6042978	.0511419	.5119336	.7133265
corr(exp,wks)	-.2976598	.1000254	-.4792842	-.0915878
corr(exp,_cons)	.0036853	.0859701	-.1633389	.1705042
corr(wks,_cons)	-.4890482	.0835945	-.6352413	-.3090207
sd(Residual)	.1319489	.0017964	.1284745	.1355172

LR test vs. linear regression:                      chi2(6) = 5006.75                      Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

## Two-way Random Effects

- Have individual-specific intercepts and time-specific intercept that are both random

- ▶  $y_{it} = \alpha_i + \delta_t + \mathbf{x}'_{it}\boldsymbol{\beta}_i + \varepsilon_{it}$
- ▶  $\alpha_i$  iid  $(\alpha, \sigma_\alpha^2)$  and  $\delta_t$  iid  $(0, \sigma_\delta^2)$ .

\* Two-way random-effects model estimated using `xtmixed`  
`xtmixed l wage exp exp2 wks ed || _all: R.t || id: , mle`

## 8. Clustered Data

- **xt** commands are not just for panel data
  - ▶ can apply several of them to clustered cross-section data.
- Consider data on individual  $i$  in village  $j$  with **clustering on village**.
- A **cluster-specific model** (here village-specific) specifies

$$y_{ji} = \alpha_j + \mathbf{x}'_{ji}\boldsymbol{\beta} + \varepsilon_{ji}.$$

- Here clustering is on village (not individual) and the repeated measures are over individuals (not time).
- Assuming **equicorrelated errors** can be more reasonable here than with panel data (where correlation dampens over time).
  - ▶ So perhaps less need for `vce(robust)` after `xtreg`.

# Estimators for Clustered Data

- First use `xtset village person` (versus `xtset id t` for panel).
- If  $\alpha_i$  is **random** use:
  - ▶ `regress` with option `vce(cluster village)`
  - ▶ `xtreg, re`
  - ▶ `xtgee` with option `exchangeable`
  - ▶ `xtmixed` for Stata 10-12 for richer models of error structure.
  - ▶ `mixed` for Stata 13 for richer models of error structure.
- If  $\alpha_i$  is **fixed** use:
  - ▶ `xtreg, fe`

## 9. Summary

- More linear panel estimators for short panels

<b>Panel IV</b>	<code>xtivreg</code>
<b>Hausman-Taylor</b>	<code>xthtaylor</code>
<b>Dynamic FE models</b>	<code>xtabond</code> , <code>xtdpdsys</code> , <code>xtdpd</code> <code>xtabond2</code> , <code>pvar</code> (user-written),
<b>Random slopes</b>	<code>xtmixed</code> ; <code>quadchk</code> ; <code>xtrc</code> <code>mixed</code> (from Stata 13 on)

- And many panel commands can be used for clustered data
  - ▶ e.g. `cluster` on `village`.

## 10. References

- Arellano, M., and S. Bond (1991), "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *Review of Economic Studies*, 58, 277-298.
- Holtz-Eakin, D., W. Newey, and H.S. Rosen (1988), "Estimating Vector Autoregressions with Panel Data," *Econometrica*, 56, 1371-1395.
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